

Introduction

The **Algebra Success Keys (ASK)** guide is a compilation of research-based practices developed to help teach algebra to students with diverse learning needs. A foundation in algebraic thinking begins in grades K-5 and continues through the middle school grades to high school algebra. It is important to recognize that in the development of algebraic thinking, students should be able to understand abstract concepts, if the concepts are developed gradually from concrete to semi-concrete or representational to abstract over a period of time (Miller & Mercer, 1993).

The purpose of the **Algebra Success Keys (ASK)** is to provide algebra teachers of grades K-12 with research-based strategies in instruction, assessments, accommodations, and technology to help students learn algebraic thinking. Math teachers and special education teachers should use this guide to enhance their teaching of algebraic thinking skills to students with diverse learning needs. This guide should be used in conjunction with action research. Action research allows a teacher to understand and reflect on what they are learning as well as how and what the students are learning.

The **Algebra Success Keys (ASK)** is not a curriculum. It is a resource available to teachers to improve student learning of algebra in grades K-12. **ASK** should be used to help teachers teach algebraic thinking using three levels of learning (Concrete-Representational-Abstract) and to meet the needs of individual students. It is based on selected “big ideas” taken from the (National Council for Teachers of Mathematics) Standards, Florida Sunshine State Standards, and research related to the teaching of algebraic thinking. (Miller & Mercer, 1993; Witzel, Mercer, & Miller, 2003).

The **Algebra Success Keys (ASK)** provides strategies for the teaching of algebra within the four standards:

- ❖ understanding patterns, relations, and functions
- ❖ representing and analyzing mathematical situations and structures using algebraic symbols
- ❖ using mathematical models to represent and understand quantitative relationships
- ❖ analyzing change in various contexts (NCTM, 2001)

This guide is divided into ten sections and includes topics such as the levels of learning, instructional strategies, assessments, accommodations and technology. **ASK** provides teachers with assistance, to enhance their instruction and their students learning environment, which will enable them to reach diverse learners in algebraic thinking.

Fundamental Components of Algebraic Thinking

It is important to understand the “big ideas” taught in algebraic thinking at each grade level. These ideas are taught in order to build on the student’s prior knowledge. High expectations are essential to helping students learn algebra at all grade levels. The NCTM Algebra Standards and the Florida Sunshine State Standards (SSS) provide a framework based on standards at each grade level in algebraic thinking. The following table aligns the four NCTM (2000) algebra standards to the SSS by grade level. Vertical understanding of benchmarks in algebraic thinking will help teachers develop concepts and strategies used to expand each student’s ability. Assessment strategies help teachers to evaluate students’ knowledge of algebraic thinking in order to differentiate instruction. The ability to identify student learning gains compared with the benchmarks provide a base for teachers to develop strategies, incorporate accommodations, and use effective technological supplements to enhance student growth.

Alignment of Sunshine State Standards and NCTM Standards

Understanding patterns, relationships, and functions		
Grades	Sunshine State Standards	NCTM 2000 Algebra Standard Expectation
K-2	<p>MA.D.1.1.1-Describes as wide variety of classification schemes and patterns related to physical characteristics and sensory attributes, such as rhythm, sound, shapes, colors, numbers, similar objects, similar event.</p> <p>MA.D.1.1.2-Recognizes, extends, generalizes, and creates a wide variety of patterns and relationships using symbols and objects.</p>	<p>Sort, classify, and order objects according to various characteristics such as size, number and other properties.</p> <p>Recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another.</p> <p>Analyze how both repeating and growing patterns are generated.</p>
3-5	<p>MA.D.1.2.1-Describes a wide variety of patterns and relationships through models, such as manipulatives, tables, graphs, rules using algebraic symbols.</p> <p>MA.D.1.2. 2-Generalizes a pattern, relation, or function to explain how a change in quantity results in a change in another.</p>	<p>Learn to represent patterns numerically, graphically, verbally and symbolically.</p> <p>Identify relationships in numerical and geometric patterns and analyze how patterns grow and change.</p> <p>Make and explain generalizations about patterns and show relationships in patterns to make predictions using tables, charts, physical objects, and symbols.</p>
6-8	<p>MA.D.1.3.1-Describes a wide variety of patterns, relationships, and functions through models, such as manipulatives, tables, graphs, expressions, equations, and inequalities.</p> <p>MA.D.1.3.2-Creates and interprets tables, graphs, equations, and verbal descriptions to explain cause-and-effect relationships.</p>	<p>Explore patterns expressed in tables, graphs, words, or symbols with an emphasis on patterns that exhibit linear relationships (constant rate of change).</p> <p>Learn to relate symbolic and graphical representations and develop an understanding of the significance of slope and y-intercept.</p> <p>Explore "What if?" questions to investigate how patterns change.</p> <p>Distinguish linear from non-linear patterns.</p>
9-12	<p>MA.D.1.4.1-Describes, analyzes, and generalizes relationships, patterns, and functions using words, symbols, variables, tables and graphs.</p> <p>MA.D.1.4.2-Determines the impact when changing parameters of given functions.</p>	<p>Create and use tables, symbols, graphs, and verbal representations to generalize and analyze patterns, relationships, and functions with increasing sophistication, and convert flexibly among various representations.</p> <p>Compare and contrast situations modeled by different types of functions, and they develop an understanding of classes of functions, both linear and nonlinear, and their properties.</p> <p>Understand functions of more than one variable, and learn to perform transformations such as composing and inverting commonly used functions.</p>

Representing and analyzing mathematical situations and structures using algebraic symbols

Grades	Sunshine State Standards	NCTM 2000 Algebra Standard Expectation: All students should:
K-2	<p>MA.D.1.1. 2-Recognizes, extends, generalizes, and creates a wide variety of patterns and relationships using symbols and objects.</p> <p>MA.D.2.1. 1-Understands that geometric symbols can be used to represent unknown quantities in expressions, equations, and inequalities.</p>	<p>Illustrate general mathematical properties of operations (i.e. the commutative property of addition) with objects or specific numbers.</p> <p>Use objects, pictures, words, or symbols to represent mathematical ideas and relationships, including the relationship of equality, and to solve problems.</p> <p>Recognize equivalent representations and expand their ability to use symbols to communicate their ideas by describing and representing quantities in different ways.</p>
3-5	<p>MA.D.1.2.2-Generalizes a pattern, relation, or function to explain how a change in quantity results in a change in another.</p> <p>MA.D.2.2.1-Represents a given simple problem situation using diagrams, models, and symbolic expressions translated from verbal phrases, or verbal phrases translated from symbolic expressions, etc.</p>	<p>Investigate, represent, describe, and explain mathematical properties, and they begin to generalize relationships and to use them in computing with whole numbers.</p> <p>Develop notions of the idea and usefulness of variables, which they may express with a box, letter, or other symbol to signify the idea of a variable as a placeholder.</p> <p>Learn to use variables to describe a rule that relates two quantities or to express relationships using equations.</p>
6-8	<p>MA.D.1.3.2-Creates and interprets tables, graphs, equations, and verbal descriptions to explain cause-and-effect relationships.</p> <p>MA.D.2.3.1-Represents and solves real-world problems graphically, with algebraic expressions, equations, and inequalities.</p> <p>MA.D.2.3.2-Uses algebraic problem-solving strategies to solve real-world problems involving linear equations and inequalities.</p>	<p>Encounter additional uses of variables as changing quantities in generalized patterns, formulas, identities, expressions of mathematical properties, equations, and inequalities.</p> <p>Explore notions of dependence and independence as variables change in relation to one another, and they develop facility in recognizing the equivalence of mathematical representations, which they can use to transform expressions; to solve problems; and to relate graphical, tabular, and symbolic representations.</p> <p>Acquire greater facility with linear equations and demonstrate how the values of slope and y-intercept affect the line.</p>
9-12	<p>MA.D.1.4.2-Determines the impact when changing parameters of given functions.</p> <p>MA.D.2.4.1 Represents real-world problem situations using finite graphs, matrices, sequences, series, and recursive relations.</p>	<p>Understand and write the meaning of equivalent forms of expressions, equations, inequalities, and relations;</p> <p>Use symbolic algebra to represent and explain mathematical relationships;</p> <p>Use a variety of symbolic representations, including recursive and parametric equations, for functions and relations.</p>

Using mathematical models to represent and understand quantitative relationships		
Grades	Sunshine State Standards	NCTM 2000 Algebra Standard Expectation: All students should:
K-2	MA.D.2.1. 2-Uses informal methods to solve real-world problems requiring simple equations that contain one variable.	Learn to use objects or pictures, and, eventually, symbols to enact stories or model situations that involve addition or subtraction of whole numbers.
3-5	MA.D.1.2.2-Generalizes a pattern, relation, or function to explain how a change in quantity results in a change in another. MA.D.2.2.2-Uses informal methods, such as physical models and graphs, to solve real-world problems involving equations and inequalities.	Realize that mathematics can be used to model numerical and geometric patterns, scientific experiments, and other physical situations, and they discover that mathematical models have the power to predict as well as describe. Employ graphs, tables, and equations to represent relationships and use their models to draw conclusions. Compare various models and investigate whether different models of a particular situation yield the same results.
6-8	MA.D.1.3.2-Creates and interprets tables, graphs, equations, and verbal descriptions to explain cause-and-effect relationships. MA.D.2.3.1-Represents and solves real-world problems graphically, with algebraic expressions, equations, and inequalities.	Use various representations, such as graphs, tables, and equations, to solve conceptualized problems that can be modeled. With the help of technology, students learn to use functions to model patterns of change, including situations in which they generate and represent real data. Explore examples of nonlinear relationships, and use a model to develop and test conjectures but the emphasis is on various contexts that are modeled by linear relationships.
9-12	MA.D.2.4.1- Represents real-world problem situations using finite graphs, matrices, sequences, series, and recursive relations. MA.D.2.4.2-Uses systems of equations and inequalities to solve real-world problems graphically, algebraically, and with matrices.	Develop skill in identifying essential quantitative relationships in a situation and in determining the type of function with which to model the relationship. Use symbolic expressions to represent relationships arising from various contexts, including situations in which they generate and use data. Using their models, students conjecture about relationships, formulate and test hypotheses, and draw conclusions about the situations being modeled.

Analyze change in various contexts		
Grades	Sunshine State Standards	NCTM 2000 Algebra Standard Expectation: All students should:
K-2	MA.D.1.1.2-Recognizes, extends, generalizes, and creates a wide variety of patterns and relationships using symbols and objects.	Describe qualitative change, such as a student's growing taller. Describe quantitative change, such as a student's growing two inches in one year.
3-5	MA.D.1.2.2-Generalizes a pattern, relation, or function to explain how a change in quantity results in a change in another.	Investigate how a change in one variable relates to a change in a second variable. Identify and describe situations with constant or varying rates of change and compare them.
6-8	MA.D.2.3.1-Represents and solves real-world problems graphically, with algebraic expressions, equations, and inequalities.	Use graphs to analyze the nature of changes in quantities in linear relationships.
9-12	MA.D.2.4.2-Uses systems of equations and inequalities to solve real-world problems graphically, algebraically, and with matrices.	Approximate and interpret rates of change from graphical and numerical data.