## Task Analysis for Algebraic Thinking by Grade Level

The following contains the major skills and concepts that support and help develop algebraic thinking for each grade level (K-12). Depending on the grade level, these skills and concepts were subdivided in some or all of the following major areas: pre-number concepts, numeration, measurement, addition concept and skills, addition computation, subtraction concepts and skills, subtraction computation, properties, variables, expressions, equations and functions, tables and graphs, fraction concepts, decimal concepts, multiplication concepts, multiplication computation, division concepts, and division computation. Furthermore, each of the tables contains the following areas: which representation level should be included (concrete, pictorial and/or abstract), type of validation the student should provide, indication of student's mastery or non-mastery of the skill or concept after proper instruction and assessment, and students' retention or non-retention of the skill or concept after proper instruction and assessment.

| Kindergarten Algebraic Thinking Checklist | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| **A. Pre-number Concepts** | | | | | | | | | |
| 1. Sorts or classifies objects by one attribute (size number and other properties) | x | | | | x | | | | |
| 2. Labels sets with one attribute | x | | | | x | | | | |
| 3. Sorts and then resorts by one attribute | x | | | | x | | | | |
| 4. Recognizes and matches patterns such as sequences of sounds, shapes or simple numbers | x | x | | | x | | | | |
| 5. Extends one attribute patterns | x | x | | | x | | | | |
| 6. Translates one attribute patterns (by changing values of an attribute, by changing attributes, or by changing instructional level) | x | x | | | x | | | | |
| 7. Creates one attribute patterns | x | | | | x | | | | |
| **B. Numeration Ideas** | | | | | | | | | |
| 1. Rote counts from 1 to a given number (up to 10) | | | x | x | | | | | |
| 2. Uses one-to-one correspondence (one number name for each object) | x | | | x | | | | | |
| 3. Counts rationally (meaningfully) from 1 to a given number (up to 10) (makes a set to match a number) | x | | | | x | | | | |
| 4. Uses the concept of zero as a possible value | x | | | | x | | | | |
| 5. Recognizes and names numerals (at least to 5) | | | x | x | | | | | |
| 6. Orders or sequences events, ideas or shapes | x | | | x | | | | | |
| 7. Orders numbers (0-20) | x | | | | x | | | | |
| 8. Uses ordinal numbers (first, last) | x | | | | x | | | | |
| 9. Compares two numbers (no numerals, up to 5) | x | | | x | | | | | |
| 10. Uses vocabulary properly: more, fewer, most, fewest | x | | | | x | | | | |
| 11. Makes a number with more or less (no numerals, up to 5) | x | | | | x | | | | |
| 12. Compares two numbers given verbally (no numerals, up to 5) | x | | | x | | | | | |
| 13. Makes two sets equal (no numerals, up to 5) | x | | | | x | | | | |
| 14. Makes two sets equal (with numerals, up to 5) | x | | | | x | | | | |
| 15. Matches sets to numerals (up to 5) | x | | | | x | | | | |

| Kindergarten Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| **C. Fraction Concepts** | | | | | | | | | |
| 1. Identifies fractions as part of a whole (rectangular shape): half | x | | | | | x | | | |
| 2. Represents fraction as part of a whole: half | x | | | | | | x | | |
| **D. Measurement** | | | | | | | | | |
| 1. Compares, sorts, orders using length and weight of objects: small-large, smallest-largest, short-tall, shortest-tallest, tall-taller, short-shorter, long-longest, farther-farthest, near-nearest, light-heavy, lightest-heaviest | x | | | | x | | | | |
| 2. Identifies time relationships: quicker, slower, sequencing events | | | x | x | | | | | |
| 3. Explores the use of the calendar to the week, day | | | x | x | | | | | |

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| **E.** | **Addition Concepts and Skills** | | | | | | | | | |
| 1. | Uses numerals to construct addition number sentences (one-digit numbers, sums up to 5) | x | | | | x | | | | |
| 2. | Solves addition exercises (sums up to 5) | x | | | | x | | | | |
| 3. | Solve addition word problems (no variables, sums up to 5) | x | | | | x | | | | |
| 4. | Explores the idea of missing addends (sums to 5) | x | | | | x | | | | |
| 5. | Explores the idea equalities (sums up to 5) | x | | | | x | | | | |
| **F.** | **Variables, Expressions, Equations, Functions** | | | | | | | | | |
| 1. | Uses a geometric symbol to represent an unknown quantity (whole numbers, to 5) | x | | | x | | | | | |
| 2. | Uses a geometric symbol to represent unknowns in expressions or equations (5 as a domain) | x | | | x | | | | | |
| 3. | Solves addition equations with one variable: $3 + 2 = \sigma, \sigma = 5$; or $0 + 2 = \lambda, \lambda = 2$. | x | | | | x | | | | |
| 4. | Solves addition word problems requiring one variable (expressions or equations, sums up to 5) | x | | | | x | | | | |
| 5. | Explores the idea that the same variables have the same value in an equation (sums up to 5): $\lambda + \lambda = 4, \lambda = 2$; or $\sigma + 0 = \sigma$ | x | | | | x | | | | |
| 6. | Explores the idea of missing addends with one variable (sums up to 5) | x | | | | x | | | | |
| 7. | Explores the idea of equalities with one variable (sums up to 5) | x | | | | x | | | | |
| **G.** | **Number Properties** | | | | | | | | | |
| 1. | Explores the addition identity element property (sums up to 5) | x | | | | x | | | | |
| 2. | Explores the addition commutative property (sums up to 5) | x | | | | x | | | | |
| 3. | Explores the idea of function involving addition and one variable (sums up to 5) | x | | | | x | | | | |

| First Grade Algebraic Thinking Checklist | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| **A. Pre-number Concepts** | | | | | | | | | |
| 1. Sorts or classifies objects by one or two attributes (size, number, or other properties) | x | x | | | x | | | | |
| 2. Labels sets with one or two attributes | x | x | | | x | | | | |
| Sorts and then resort by one or two attributes | x | x | | | x | | | | |
| 3. Recognizes and matches patterns such as sequences of sounds, shapes or numbers | x | x | x | | x | | | | |
| 4. Extends one attribute patterns | x | x | x | | x | | | | |
| 5. Translates one attribute patterns from one representation to another | x | x | x | | x | | | | |
| 6. Creates one attribute patterns | x | x | x | | x | | | | |
| **B. Numeration Ideas** | | | | | | | | | |
| 1. Rote counts from 1 to a given number (to 100) | | | x | x | | | | | |
| 2. Rote counts from a given number to (not 1) to another given number | x | | | | x | | | | |
| 3. Counts backwards | | | x | x | | | | | |
| 4. Rote skip counts by 10s | | | x | x | | | | | |
| 5. Rote skip counts by 5s | | | x | x | | | | | |
| 6. Rote skip counts by 2s | | | x | x | | | | | |
| 7. Uses one-to-one correspondence (one number name for each object to 100) | x | x | | | x | | | | |
| 8. Counts rationally (meaningfully) to 100 (make a set to match a number) | x | x | | x | x | | | | |
| 9. Skip counts rationally by 10s | x | x | | x | x | | | | |
| 10. Skip counts rationally by 5s | x | x | | x | x | | | | |
| 11. Skip counts rationally by 2s | x | x | | x | x | | | | |
| 12. Supplies the missing number: 20, 21, ___, 22; or 3, 7, ___, 11 | | | x | x | x | | | | |
| 13. Orders three or more given sets | x | x | | | x | | | | |
| 14. Uses the concept of zero as a possible value | x | x | | | x | | | | |
| 15. Orders numbers (0-20) | x | x | | | x | | | | |
| 16. Uses before and after properly | x | | | x | | | | | |
| 17. Matches sets to numerals (up to 10) | x | x | | x | x | | | | |
| 18. States the relationship between numbers: >, <, = | x | x | x | x | x | | | | |
| 19. Uses ordinal numbers (first, second, third, fourth, fifth) | x | x | | x | x | | | | |
| **C. Measurement** | | | | | | | | | |
| 1. Compares, sorts, orders using length and weight of objects: small-large, smallest-largest, short-tall, shortest-tallest, tall-taller, short-shorter, long-longest, farther-farthest, near-nearest, light-heavy, lightest-heaviest, shorter, longer, same | x | x | | x | x | | | | |
| 2. Uses arbitrary units to measure | x | x | | x | x | | | | |

| First Grade Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 3. Uses metric units to measure: centimeter, liter, kilogram | x | x | | x | x | | | | |
| 4. Uses standard units to measure: inch, cup, pint, | x | x | | x | x | | | | |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| quart, pound | | | | | | | | | | |
| 5. Identifies time relationships: quicker, slower, sequencing events | | | x | x | | | | | | |
| 6. Uses the calendar to the week, day, month | | | x | x | | | | | | |
| 7. Reads numerals on clock face | | | x | x | | | | | | |
| 8. Reads ___:00 as o'clock | | | x | x | | | | | | |
| 9. Shows time to the hour | x | | | | x | | | | | |
| 10. Reads ___:30 as half pass ___ and ___-thirty | | | x | x | | | | | | |
| 11. Shows time to the half-hour | x | | | | x | | | | | |
| 12. Recognizes penny, nickel, dime | x | | | x | x | | | | | |
| 13. Counts pennies, nickels | x | | | | x | | | | | |
| 14. Makes purchases to 25¢ | x | | | | x | | | | | |
| **D. Extending Numeration Ideas: Place Value** | | | | | | | | | | |
| 1. Uses place value through 99 (tens and ones) | x | x | x | x | x | | | | | |
| 2. Recognizes and names numerals (at least to 10) | | | x | x | | | | | | |
| 3. Orders or sequence events, ideas or shapes | x | x | x | x | | | | | | |
| 4. Uses vocabulary properly: more, fewer, most, fewest | x | x | | x | x | | | | | |
| 5. Compares two given numbers (with numerals, up to 10) | x | x | x | x | x | | | | | |
| 6. Makes a number with more or less (no numerals, up to 10) | x | x | | x | x | | | | | |
| 7. Makes a number with more or less (with numerals, up to 10) | x | x | | x | x | | | | | |
| 8. Compares two numbers given verbally (no numerals, up to 10) | x | x | | x | x | | | | | |
| 9. Makes two sets equal (no numerals, up to 10) | x | | | | x | | | | | |
| 10. Makes two sets equal (with numerals, up to 10) | x | | | | x | | | | | |
| **E. Addition Concepts and Skills** | | | | | | | | | | |
| 1. Uses numerals to construct addition number sentences (one-digit numbers, sums up to 10) | x | | | | x | | | | | |
| 2. Solves addition problems (addition concept, sums up to 10, no variables) | x | x | x | x | x | | | | | |
| 3. Solves column addition problems (3 addends, no regrouping) | x | | | | x | | | | | |
| 4. Recites basic addition facts (sums to 10) | | | x | x | | | | | | |
| 5. Explores the idea of equality (no variables, sums to 10) | x | | | | x | | | | | |
| 6. Explores the idea of missing addends (no variables, sums to 10) | x | | | | x | | | | | |

| First Grade Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| **F. Subtraction Concepts and Skills** | | | | | | | | | |
| 1. Solves subtraction problems (subtraction take-away concept, differences from 2 - 10) | x | x | x | x | x | | | | |
| 2. Solves subtraction word problems (requiring no variables, for differences 10 or less) | x | x | x | x | x | | | | |
| 3. Recites basic subtraction facts (for differences 10 or less) | | | x | x | | | | | |
| 4. Explores the relationship between addition and subtraction (addition and subtraction are inverse operations) | x | | | | x | | | | |
| **G. Addition Computation** | | | | | | | | | |
| 1. Adds tens (40 + 30) | x | | | | x | | | | |
| 2. Adds a two-digit and a one-digit number (no regrouping) | x | | | | x | | | | |
| 3. Adds two two-digit numbers (no regrouping) | x | | | | x | | | | |
| 4. Matches a given story to a graph (two variables) | x | | | | x | | | | |
| **H. Number Properties** | | | | | | | | | |
| 1. Explores the addition commutative property (sums up to 10) | x | x | | | x | | | | |
| 2. Explores the addition identity element property (sums up to 10) | x | x | | | x | | | | |
| **I. Variables, Expressions, Equations, Functions** | | | | | | | | | |
| 1. Uses a geometric symbols ($\nu$,$\sigma$, or $\lambda$) to represent an unknown quantity (whole numbers with 10 as domain) | | | x | | x | | | | |
| 2. Uses a geometric symbol to represent an unknown in expressions or equations | x | | | x | | | | | |
| 3. Uses the same value for the same variable in equations (sums up to 10): $\lambda + \lambda = 4$, $\lambda = 2$; or $\sigma + 0 = \sigma$ | x | | | | x | | | | |
| 4. Uses the same or different values for different variables in equations (one or two variables, sums up to 10): $\lambda + \sigma = 4$, $\lambda$ and $\sigma = 2$; or $\sigma + \lambda = 3$, $\sigma = 1$ and $\lambda = 2$ | x | | | | x | | | | |
| 5. Solves subtraction word problems requiring one variable (expressions or equations, up to ten) | x | x | x | x | x | | | | |
| 6. Solves addition equations with one variable: $3 + 2 = \sigma$, $\sigma = 5$; or $0 + 2 = \lambda$, $\lambda = 2$ | x | x | | x | x | | | | |
| 7. Solves addition word problems requiring one variable (expressions or equations, up to ten) | x | x | x | x | x | | | | |
| 8. Explores the idea of function involving addition, missing addends, and one variable (sums to 10) | x | | | | x | | | | |
| 9. Explores the idea of missing addends (one or two variables, sums to 10) | x | | | | x | | | | |

| First Grade Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 10, Explores the idea of equality (one variable, sums up to 10) | x | | | | x | | | | |
| **J. Tables and Graphs** | | | | | | | | | |
| 1. Reads, locates and interprets information from a table: important terms: row, column, top, bottom, first, ordinal numbers | x | x | | | x | | | | |
| 2. Makes a table to organize and solve problems | x | x | | | x | | | | |
| 3. Uses a number line involving whole numbers | x | | | x | | | | | |
| 4. Graphs data | x | x | | | x | | | | |
| **K. Fractions** | | | | | | | | | |
| 1. Identifies fractions as part of a whole (rectangular shape): 1/2, 1/3, 1/4 | x | x | | x | | | | | |
| 2. Represents fractions as part of a whole: 1/2, 1/3, 1/4 | x | x | | | x | | | | |

| Second Grade Algebraic Thinking Checklist | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| **A. Pre-number Concepts** | | | | | | | | | |
| 1. orts or classify objects by one or two attributes | x | x | | | x | | | | |
| 2. Labels sets with one or two attributes | x | x | | | x | | | | |
| 3. Sorts and then resort by one or two attributes | x | x | | | x | | | | |
| 4. Matches one, two or three attribute patterns | x | x | x | | x | | | | |
| 5. Extends one, two or three attribute patterns | x | x | x | | x | | | | |
| 6. Translates one, two and three attribute patterns | x | x | x | | x | | | | |
| 7. Creates one, two or three attribute patterns | x | x | x | | x | | | | |
| **B. Numeration Ideas** | | | | | | | | | |
| 1. Rote counts from 1 to a given number (to 999) | | | x | x | | | | | |
| 2. Rote counts from a given number to (not 1) to another given number | x | | | | x | | | | |
| 3. Counts backwards | | | x | x | | | | | |
| 4. Rote skip counts by 10s | | | x | x | | | | | |
| 5. Rote skip counts by 5s | | | x | x | | | | | |
| 6. Rote skip counts by 2s | | | x | x | | | | | |
| 7. Rote skip counts by 3s | | | x | x | | | | | |
| 8. Uses one-to-one correspondence (one number name for each object to 100) | x | x | | | x | | | | |
| 9. Uses one-to-many correspondence (one number name for each group of objects to 10) | x | x | | | x | | | | |
| 10. Counts rationally (meaningfully) to 999 (make a set to match a number) | x | x | | x | x | | | | |
| 11. Skip counts rationally by 10s | x | x | | x | x | | | | |
| 12. Skip counts rationally by 5s | x | x | | x | x | | | | |
| 13. Skip counts rationally by 2s | x | x | | x | x | | | | |
| 14. Skip counts rationally by 3s | x | x | | x | x | | | | |
| 15. Supplies the missing number: 20, 21, ___, 22; or 3, 7, ___, 11. | | | x | x | x | | | | |
| 16. Orders three or more given sets | x | x | | | x | | | | |
| 17. Uses the concept of zero as a possible value | x | x | | | x | | | | |
| 18. Orders numbers (0-20) | x | x | | | x | | | | |
| 19. Uses before and after properly | x | | | x | | | | | |
| 20. Matches sets to numerals (up to 10) | x | x | | x | x | | | | |
| 21. States the relationship between numbers | x | x | x | x | x | | | | |
| 22. Identifies odd and even numbers | x | x | x | x | x | | | | |
| 23. Uses ordinal numbers (first, second, third, fourth, fifth) | x | x | | x | x | | | | |
| **C. Measurement** | | | | | | | | | |
| 1. Uses arbitrary units to measure | x | x | | x | x | | | | |
| 2. Compares, sorts, orders using length weight and volume | x | x | x | x | x | | | | |

| Second Grade Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 3. Uses metric units to measure: centimeter, liter, kilogram, gram | x | x | | x | x | | | | |
| 4. Uses standard units to measure: inch, half-inch, foot, cup, pint, gallon, quart, pound | x | x | | x | x | | | | |
| 5. Explores the idea of temperature (introduction) | x | | | | x | | | | |
| 6. Identifies time relationships: quicker, slower, sequencing events | x | | | | x | | | | |
| 7. Uses of the calendar to the week, day | | | x | x | | | | | |
| 8. Reads numerals on clock face | | | x | x | | | | | |
| 9. Reads ___:00 as o'clock | | | x | x | | | | | |
| 10. Shows time to the hour | x | | | | x | | | | |
| 11. Reads ___:30 as half pass ___ and ___-thirty | | | x | x | | | | | |
| 12. Shows time to the half-hour | x | | | | x | | | | |
| 13. Reads 5-, 10-, and 15-minutes multiples after the hour | | | x | x | | | | | |
| 14. Shows time in 5-, 10-, and 15-minutes multiple intervals | x | | | | x | | | | |
| 15. Reads time to the minute | | | x | x | | | | | |
| 16. Shows time to the minute | x | x | | | x | | | | |
| 17. Recognizes and show time after the hour, before the hour | x | x | | | | | | | |
| 18. Recognizes penny, nickel, dime, quarter, dollar | x | x | | x | x | | | | |
| 19. Counts pennies, nickels, dime, from largest to smallest amounts | x | x | | x | x | | | | |
| 20. Makes purchases to 25¢, 50¢, $1.00 | x | x | | x | x | | | | |
| **D. Extending Numeration Ideas: Place Value** | | | | | | | | | |
| 1. Uses place value through 999 (hundreds, tens and ones) | x | x | x | x | x | | | | |
| 2. Recognizes and names numerals at least to 100 | | | x | x | | | | | |
| 3. Orders or sequence events, ideas or shapes | x | x | x | x | | | | | |
| 4. Uses vocabulary properly: more, fewer, most, fewest, one more, one less | x | x | | x | x | | | | |
| 5. Compares two given numbers (with numerals, using =, <, > symbols, up to 100) | x | x | x | x | x | | | | |
| 6. Makes a number with more or less (no numerals, up to 100) | x | x | x | x | x | | | | |
| 7. Makes a number with more or less (with or without numerals, up to 100) | x | x | x | x | x | | | | |
| 8. Compares two numbers given verbally (with or without numerals, up to 100) | x | x | x | x | x | | | | |
| 9. Makes two sets equal (with or without numerals, up to 100) | x | x | x | x | x | | | | |

| Second Grade Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 10. Makes two sets equal (with or without numerals, up to 100) | x | x | x | | x | | | | |
| 11. Uses numerals to construct addition number sentences (one-digit numbers, sums up to 100) | x | x | x | x | x | | | | |
| **E. Addition Concepts and Skills** | | | | | | | | | |
| 1. Solves addition problems (addition concept, sums up to 18, no variables) | x | x | x | x | x | | | | |
| 2. Solves column addition problems (3 addends, with or without regrouping) | x | x | x | x | x | | | | |
| 3. Recites basic addition facts (sums to 18) | | | x | x | | | | | |
| 4. Uses the idea of missing addends (no variables, sums to 18) | x | | | | x | | | | |
| 5. Uses the idea of equality (without variables): 4 + 1 = 3 + 2 | x | x | x | x | x | | | | |
| **F. Subtraction Concepts and Skills** | | | | | | | | | |
| 1. Solves subtraction problems (take-away and comparison concepts, differences 0 - 18) | x | x | x | x | x | | | | |
| 2. Recites basic subtraction facts (differences from 0 - 18) | | | x | x | | | | | |
| 3. Solves subtraction word problems (requiring no variables, differences from 0 - 18) | x | x | x | x | x | | | | |
| 4. Solves subtraction word problems requiring one or two variables (expressions or equations, differences from 0 - 18) | x | x | x | x | x | | | | |
| **G. Addition Computation** | | | | | | | | | |
| 1. Adds multiples of ten (no regrouping) | x | x | | | x | | | | |
| 2. Adds multiples of ten (with regrouping) | x | x | | | x | | | | |
| 3. Adds multiples of hundred (no regrouping) | x | x | | | x | | | | |
| 4. Adds a two-digit and a one-digit number (no regrouping) | x | x | | | x | | | | |
| 5. Adds a two-digit and a one-digit number (with regrouping) | x | x | | | x | | | | |
| 6. Adds two two-digit numbers (no regrouping) | x | x | | | x | | | | |
| 7. Adds two two-digit numbers (with regrouping) | x | x | | | x | | | | |
| **H. Subtraction Concepts and Skills** | | | | | | | | | |
| 1. Subtracts a one-digit from a two-digit (no regrouping) | x | | | | x | | | | |
| 2. Subtracts two-digit numbers (no regrouping) | x | | | | x | | | | |
| 3. Subtract a one-digit from a two-digit (with regrouping) | x | | | | x | | | | |
| 4. Subtracts two-digit numbers (with regrouping) | x | | | | x | | | | |

| Second Grade Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| **I. Variables, Expressions, Equations, Functions** | | | | | | | | | |
| 1. Uses a geometric symbols ($\nu$,$\sigma$, or $\lambda$) to represent an unknown quantity (whole numbers with 18 as domain) | | | x | | x | | | | |
| 2. Uses a geometric symbol to represent an unknown in expressions or equations | x | x | x | x | x | | | | |
| 3. Uses the same value for the same variable in equations (sums up to 18): $\lambda + \lambda = 4$, $\lambda = 2$; or $\sigma + 0 = \sigma$ | x | x | x | x | x | | | | |
| 4. Uses the same or different values for different variables in equations (one, two or three variables, sums up to 18): $\lambda + \sigma = 4$, $\lambda$ and $\sigma = 2$; $\sigma + \lambda = 3$, $\sigma = 1$ and $\lambda = 2$; or $\sigma + \lambda + \nu = 3$, $\sigma$, $\nu$ and $\lambda = 1$ | x | x | x | x | x | | | | |
| 5. Solves addition word problems requiring one variable (expressions or equations, up to 18) | x | x | x | x | x | | | | |
| 6. Explores the idea of balancing scales involving addition and missing addends, and one or two variable (sums up to 18) to represent equations, and equalities | x | x | | | x | | | | |
| 7. Uses algebraic expressions to translate verbal ideas; for example, "add 5 to another number and the sum is 7" can translated as "$5 + \sigma = 7$", or write an algebraic expression to represent a given situation: weight of a box plus five, and three years less than a certain age | x | x | x | | x | | | | |
| 8. Solves addition equations with 1 or 2 variables: $3 + 2 = \sigma$, $\sigma = 5$; or $0 + 2 = \lambda$, $\lambda = 2$ | x | x | x | x | x | | | | |
| 9. Uses the idea of equality (one or two variables): $4 + \sigma = 3 + 2$, $\sigma = 1$; or $4 + \sigma = 3 + \lambda$, $\sigma = 1$ and $\lambda = 2$ | x | x | x | x | x | | | | |
| 10. Uses the idea of function involving addition and missing addends and one variable (sums up to 18) | x | x | x | x | x | | | | |
| 11. Uses the idea of missing addends (one or two variables, sums to 18) | x | x | | | x | | | | |
| 12. Finds the pattern (use objects and look for a pattern, no verbal rules) | x | x | | | x | | | | |
| 13. Finds the pattern (use objects and look for a pattern, with verbal rules) | x | x | | | x | | | | |

| Second Grade Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| **J. Number Properties** | | | | | | | | | |
| 1. Identifies and uses the addition identity element property (sums up to 18) | x | x | x | x | x | | | | |
| 2. Generalizes, identifies and uses zero as the identity element for addition using algebraic expressions: for all σ, σ + 0 = σ, σ represents any whole number from 0 to 18 | x | x | x | x | x | | | | |
| 3. Generalizes, identifies and uses the idea of the addition commutative property using variables: for any number λ and σ, λ + σ = σ + λ, σ and λ represents 0 - 18 | x | x | x | x | x | | | | |
| 4. Generalizes, identifies and uses the associative property (sums up to 18) using algebraic expressions | x | x | | | x | | | | |
| 5. Use properties to justify and manipulate equations involving two variables (commutative and identity element for addition), for example, 'Why is σ + 4 always equal to 4 + σ" | x | x | x | x | x | | | | |
| 6. Describes quantitative change, such as a student's growing taller | x | | | x | | | | | |
| 7. Describes quantitative change, such as a student's growing two inches in one year | x | | | x | | | | | |
| **K. Tables and Graphs** | | | | | | | | | |
| 1. Matches a given story to a graph (2 variables) | x | x | | | x | | | | |
| 2. Uses a number line involving whole numbers | | | | | | | | | |
| 3. Graphs data (involving the relationship between two attributes; for example, size and height) | x | x | x | | x | | | | |
| 4. Reads, locates and interprets information from a table: important terms: row, column, top, bottom, first, ordinal numbers | x | x | | | x | | | | |
| 5. Makes a table to organize and solve problems | x | x | | | x | | | | |
| 6. Explores plotting points on x- and y-axes | | | | | | | | | |
| 7. Explores the generalization for the perimeter formula for any triangle, or rectangle | | | | | | | | | |
| **L. Fractions** | | | | | | | | | |
| 1. Identifies fractions as part of a whole (rectangular shape): 1/2, 1/3, 1/4 | x | x | x | x | x | | | | |
| 2. Represents fractions as part of a whole: 1/2, 1/3, 1/4 | x | x | x | | x | | | | |
| 3. Compares fractions using <, >, = | x | | | | x | | | | |
| **M. Multiplication Concepts** | | | | | | | | | |
| 1. Explores meaning of multiplication | x | | | | x | | | | |

Grades 3-4

| Third Grade Algebraic Thinking Checklist | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| **A. Numeration Ideas** | | | | | | | | | |
| 1. Uses one-to-one correspondence (one number name for each object to 100) | x | x | | | x | | | | |
| 2. Uses one-to-many correspondence (one number name for each group of objects to 10) | x | x | | | x | | | | |
| 3. Counts rationally (meaningfully) to 100,000 (make a set to match a number) | x | x | | x | x | | | | |
| 4. Skip counts rationally by 10s | x | x | | x | x | | | | |
| 5. Skip counts rationally by 5s | x | x | | x | x | | | | |
| 6. Skip counts rationally by 2s | x | x | | x | x | | | | |
| 7. Skip counts rationally by 3s | x | x | | x | x | | | | |
| 8. Supplies the missing number: 20, 21, ___, 22; or 3, 7, ___, 11. | | | x | x | x | | | | |
| 9. Orders three or more given sets | x | x | | | x | | | | |
| 10. Uses the concept of zero as a possible value | x | x | | | x | | | | |
| 11. Compares two given numbers | x | x | x | x | x | | | | |
| 12. Rounds numbers | | | x | x | | | | | |
| 13. Orders numbers (0-100) | x | x | x | x | x | | | | |
| 14. Uses before and after properly | x | | | x | | | | | |
| 15. Matches sets to numerals (up to 100) | x | x | | x | x | | | | |
| 16. States the relationship between numbers | x | x | x | x | x | | | | |
| 17. Identifies odd and even numbers | x | x | x | x | x | | | | |
| 18. Uses ordinal numbers (first, second, third, fourth, fifth) | x | x | | x | x | | | | |
| **B. Extending Numeration Ideas: Place Value** | | | | | | | | | |
| 1. Uses place value through 999,999 (thousands, hundreds, tens, ones) | x | x | x | x | x | | | | |
| 2. Recognizes and names numerals (at least up to thousands) | | | x | x | | | | | |
| 3. Orders or sequence events, ideas or shapes | x | x | x | x | | | | | |
| 4. Uses vocabulary properly: more, fewer, most, fewest, one more, one less | x | x | | x | x | | | | |
| 5. Makes a number with more or less (with or without numerals, up to thousands) | x | x | x | x | x | | | | |
| 6. Makes two sets equal (with or without numerals, up to thousands) | x | x | x | x | x | | | | |
| 7. Makes two sets equal (with or without numerals, up to thousands) | x | x | x | | x | | | | |
| 8. Compares two numbers given verbally (with or without numerals, up to thousands) | x | x | x | x | x | | | | |
| 9. Compares two given numbers (with numerals, using =, <, > symbols, up to thousands) | x | x | x | x | x | | | | |

| Third Grade Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 10. Uses numerals to construct addition number sentences (one-digit numbers, sums up to 900) | x | x | x | x | x | | | | |
| **C. Measurement** | | | | | | | | | |
| 1. Uses metric units to measure (comparing, estimating, measuring): centimeter, meter, liter, milliliter, kilogram, gram, degrees Celsius, changing metric measures 100cm=1 m | x | x | x | x | x | | | | |
| 2. Uses standard units to measure (comparing, estimating, measuring): inch, half-inch, foot, yard, cup, pint, gallon, half-gallon, quart, pound, ounce, changing standard measures | x | x | x | x | x | | | | |
| 3. Explores the ideas of perimeter, area, volume | x | | | | x | | | | |
| 4. Uses the calendar to the week, day, month, year | | | x | x | | | | | |
| 5. Reads 5-, 10-, and 15-minutes multiples after the hour | | | x | x | | | | | |
| 6. Shows time in 5-, 10-, and 15-minutes multiple intervals | x | | | | x | | | | |
| 7. Reads time to the minute | | | x | x | | | | | |
| 8. Shows time to the minute | x | x | x | x | x | | | | |
| 9. Recognizes and show time after the hour, before the hour, fraction portions after and before the hour | x | x | x | x | x | | | | |
| 10. Recognizes penny, nickel, dime, quarter, half-dollars, dollar, other paper currency | x | | | x | | | | | |
| 11. Counts pennies, nickels, dimes, quarters, half-dollars, dollars | x | | | | x | | | | |
| 12. Counts mixed type of money from largest to smallest amounts | x | | | | x | | | | |
| 13. Makes purchases to 25¢, 50¢, $1.00, $5.00 | x | | | | x | | | | |
| 14. Makes change from 25¢, 50¢, $1.00 | x | | | | x | | | | |
| **D. Addition Concepts and Skills** | | | | | | | | | |
| 1. Solves addition problems (addition concept, sums up to 18, no variables) | x | x | x | x | x | | | | |
| 2. Recites basic addition facts (sums to 18) | | | x | x | | | | | |
| 3. Solves column addition problems (3 addends, with or without regrouping) | x | x | x | x | x | | | | |
| 4. Uses the idea of equality (without variables): 4 + 1 = 3 + 2 | x | x | x | x | x | | | | |
| 5. Uses the idea of missing addends (no variables) | x | x | x | x | x | | | | |
| 6. Adds money | x | | | | x | | | | |
| **E. Subtraction Concepts and Skills** | | | | | | | | | |
| 1. Solves subtraction problems (subtraction take away and comparison concepts, differences from 0 - 18) (knowledge) | x | x | x | x | x | | | | |

| Third Grade Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 2. Recites basic subtraction facts (differences from 0 - 18) (speed) | | | x | x | | | | | |
| 3. Solves subtraction word problems (requiring no variables, differences from 0 - 18) | x | x | x | x | x | | | | |
| **F. Addition Computation** | | | | | | | | | |
| 1. Adds multiples of ten (no regrouping) | x | x | x | | x | | | | |
| 2. Adds multiples of ten (with regrouping) | x | x | x | | x | | | | |
| 3. Adds multiples of hundred (no regrouping) | x | x | x | | x | | | | |
| 4. Adds a two-digit and a one-digit number (no regrouping) | x | x | x | | x | | | | |
| 5. Adds 2-digit and 1-digit numbers (with regrouping) | x | x | x | | x | | | | |
| 6. Adds two two-digit numbers (no regrouping) | x | x | x | | x | | | | |
| 7. Adds two two-digit numbers (with regrouping) | x | x | | | x | | | | |
| 8. Adds three digit-numbers (no regrouping) | x | x | | | x | | | | |
| 9. Adds multiples of 100 (no regrouping) | x | x | x | | x | | | | |
| 10. Adds three digit-numbers (with regrouping): in ones column, ones and tens columns, all | x | x | | | x | | | | |
| 11. Adds monetary amounts with and without regrouping | x | | | | x | | | | |
| **G. Subtraction Computation** | | | | | | | | | |
| 1. Subtracts two-digit numbers (no regrouping) | x | x | x | | x | | | | |
| 2. Subtracts a one-digit from a two-digit (with regrouping) | x | x | x | | x | | | | |
| 3. Subtracts two-digit numbers (with regrouping) | x | x | x | | x | | | | |
| 4. Explains the relationship between addition and subtraction (+ and - are inverse operations) | x | x | x | x | x | | | | |
| 5. Subtracts three-digit numbers (no regrouping) | x | | | | x | | | | |
| 6. Subtracts three- and four-digit numbers with regrouping in the ones and tens places | x | | | | x | | | | |
| 7. Subtracts a one-digit from a two-digit (no regrouping) | x | x | x | | x | | | | |
| 8. Subtracts monetary amounts with and without regrouping | x | | | | x | | | | |
| **H. Multiplication Concepts** | | | | | | | | | |
| 1. Explains the meaning of multiplication (one-digit factors) (knowledge) | x | | | | x | | | | |
| 2. Recites multiplication basic facts (0-25, speed) | | | x | x | | | | | |
| **I. Variables, Expressions, Equations, Functions** | | | | | | | | | |
| 1. Uses literal symbols (x, y, z, a, b, or others) to represent an unknown quantity, whole numbers | | | x | | x | | | | |
| 2. Uses a literal symbols to represent an unknown in expressions or equations | x | x | x | x | x | | | | |

| Third Grade Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 3. Solves addition word problems requiring one variable (expressions or equations) | x | x | x | x | x | | | | |
| 4. Uses the idea of missing addends (one or two | x | x | x | x | x | | | | |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | variables) | | | | | | | | | |
| 5. | Uses the idea of balancing scales involving addition and missing addends, and one or two variable to represent equations, and equalities | x | x | | | x | | | | |
| 6. | Uses the same value for the same variable in equations (sums up to 18): x + y = 4, y = 2; or z + 0 = z | x | x | x | x | x | | | | |
| 7. | Uses the same or different values for different variables in equations (one, two or three variables, sums up to 18): y + y = 4 and y = 2; x + y = 3, x = 1 and y = 2; or x + y + z = 3, x, y and z = 1 | x | x | x | x | x | | | | |
| 8. | Uses algebraic expressions to translate verbal ideas; for example, "add 5 to another number and the sum is 7" can translated as "5 + y = 7", or write an algebraic expression to represent a given situation: weight of a box plus five, and three years less than a certain age as w + 5 | x | x | x | | x | | | | |
| 9. | Solves addition equations with one or two variables: 3 + 2 = x , x = 5; or 0 + 2 = a, a = 2 | x | x | x | x | x | | | | |
| 10, | Uses the ideas of equality and inequalities (one, two or three variables): 4 + y = 3 + 2, y = 1; or 4 + x = 3 + y, x = 1 and y = 2 | x | x | x | x | x | | | | |
| 11. | Uses the idea of function involving addition and missing addends and one, two or three variables (sums, differences) | x | x | x | x | x | | | | |
| 12. | Explores the idea of function for multiplication and one, two or three variables (products) | x | x | x | x | x | | | | |
| 13. | Finds and extends patterns (use objects and look for a pattern, no verbal rules) | x | x | | | x | | | | |
| 14. | Finds and extends patterns (use objects and look for a pattern, with verbal rules) | x | x | | | x | | | | |
| 15. | Describes, extends and makes generalizations about geometric & numeric patterns (formula) | x | x | | | x | | | | |
| 16. | Solves subtraction word problems requiring one or two variables (expressions or equations) | x | x | x | x | x | | | | |
| 17. | Represents and analyzes patterns and functions, using words, tables, and graphs | x | x | | | x | | | | |
| 18. | Investigates how a change in one variable relates to a change in a second variable | x | | | | x | | | | |
| 19. | Investigates and describes situations with constant or varying rates of change and compares them | x | | | | x | | | | |

**Third Grade Algebraic Thinking Checklist Cont. . . .**

| Skill/Concept | Representation | | | Validation | | Mastery | | Retention | |
|---|---|---|---|---|---|---|---|---|---|
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| **J. Number Properties** | | | | | | | | | |
| 1. Generalizes, identifies and uses zero as the identity element for addition using algebraic expressions: for all x, x + 0 = x, x represents any whole number | x | x | x | x | x | | | | |
| 2. Generalizes, identifies and uses the commutative property for addition using variables: for any number a and b, a + b = b | x | x | x | x | x | | | | |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | + a , a and b represents whole numbers | | | | | | | | | |
| 3. | Generalizes, identifies and uses the associative property (sums) using algebraic expressions | x | x | | | x | | | | |
| 4. | Use properties to justify and manipulate equations involving two variables (commutative and identity element for addition), for example, 'Why is x + 4 always equal to 4 + x " | x | x | x | x | x | | | | |
| **K.** | **Tables and Graphs** | | | | | | | | | |
| 1. | Matches a given story to a graph (one, two or three variables) | x | x | | | x | | | | |
| 2. | Uses a number line involving whole numbers and fractions | x | x | x | x | x | | | | |
| 3. | Graphs data (involving the relationship between two or three attributes; for example, size, thickness and height) | x | x | x | | x | | | | |
| 4. | Reads, locates and interprets information from a table: important terms: row, column, top, bottom, first, ordinal numbers | x | x | | | x | | | | |
| 5. | Makes a table to organize and solve problems | x | x | | | x | | | | |
| 6. | Plots points on x- and y-axes | x | x | | | x | | | | |
| 7. | Expresses the generalization for the perimeter formula for any triangle, or rectangle: a + b + c = p of a triangle | | | x | x | | | | | |
| **L.** | **Fractions** | | | | | | | | | |
| 1. | Identifies fractions as part of a whole (rectangular and circular shapes): 1/2, 1/3, 1/4 | x | x | x | x | x | | | | |
| 2. | Represents fractions as part of a whole (rectangular and circular shapes): 1/2, 1/3, 1/4 | x | x | x | x | x | | | | |
| 3. | Compares fractions using <, >, = | x | x | x | x | x | | | | |
| 4. | Identifies fractions as part of a set | x | x | | x | x | | | | |
| 5. | Identifies and represent many fractions | x | x | x | x | x | | | | |
| 6. | Finds equivalent fractions | x | x | | | x | | | | |

| Fourth Grade Algebraic Thinking Checklist | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| **A. Numeration** | | | | | | | | | |
| 1. Counts rationally to 100,000,000 | x | x | x | x | x | | | | |
| 2. Compares and orders two or more numbers (using =, <, > symbols, whole numbers) | x | x | x | x | x | | | | |
| 3. Rounds numbers | | | x | x | | | | | |
| 4. Identifies odd and even numbers | x | x | x | x | x | | | | |
| 5. Uses ordinal numbers (first, second, third, fourth, fifth . . .) | x | x | | x | x | | | | |
| 6. Explores the idea of integers (as an extension of whole numbers, one-digit numbers, direction & magnitude ideas, number line, 2-color chips) | x | | | | x | | | | |
| **B. Extending Numeration Ideas: Place Value** | | | | | | | | | |
| 1. Uses place value through 999,999 (thousands, hundreds, tens, ones) | x | x | x | x | x | | | | |
| 2. Uses decimal place value to the hundredths | x | x | x | x | x | | | | |
| 3. Recognizes & names numerals (at least to 100) | | | x | x | | | | | |
| 4. Makes a number with more or less (whole numbers) | x | x | x | x | x | | | | |
| 5. Makes two sets equal (with or without numerals, whole numbers) | x | x | x | x | x | | | | |
| 6. Makes two sets equal (whole numbers) | x | x | x | | x | | | | |
| 7. Rounds numbers (whole numbers) | | | x | | x | | | | |
| 8. Estimates quantities (whole numbers) | x | x | x | x | x | | | | |
| 9. Uses numerals to construct addition number sentences (one-digit numbers, sums up to 1000) | x | x | x | x | x | | | | |
| **C. Measurement** | | | | | | | | | |
| 1. Uses metric units to measure (comparing, estimating, measuring): centimeter, meter, liter, milliliter, kilogram, gram, degrees Celsius, changing metric measures 100cm=1 m | x | x | x | x | x | | | | |
| 2. Uses standard units to measure (comparing, estimating, measuring): inch, half-inch, foot, yard, cup, pint, gallon, half-gallon, quart, pound, ounce, changing standard measures | x | x | x | x | x | | | | |
| 3. Uses the ideas of perimeter, area, volume | x | x | x | x | x | | | | |
| 4. Uses the calendar to the week, day, month, year | | | x | x | | | | | |
| 5. Reads 5-, 10-, and 15-minutes multiples after the hour | | | x | x | | | | | |
| 6. Shows time in 5-, 10-, and 15-minutes multiple intervals | x | | | | x | | | | |
| 7. Reads time to the minute | | | x | x | | | | | |
| 8. Shows time to the minute | x | x | x | x | x | | | | |
| 9. Recognizes and shows time after & before the hour, fraction portions after & before the hour | x | x | x | x | x | | | | |

| Fourth Grade Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 10. Reads time to the second by using a second hand | | | x | x | | | | | |
| 11. Identifies time zones | | | x | x | | | | | |
| 12. Recognizes penny, nickel, dime, quarter, half-dollars, dollar, other paper currency | x | x | | x | | | | | |
| 13. Counts pennies, nickels, dimes, quarters, half-dollars, dollars | x | x | | | x | | | | |
| 14. Counts mixed type of money in random order | x | x | | | x | | | | |
| 15. Makes purchases to $20.00 | x | | | | x | | | | |
| 16. Makes change to $10.00 | x | | | | x | | | | |
| **D. Fractions** | | | | | | | | | |
| 1. Identifies numerator & numerator of a fraction | | | x | x | | | | | |
| 2. Finds equivalent fractions | x | x | x | x | x | | | | |
| 3. Reads and writes mixed numbers | | | x | x | x | | | | |
| 4. Compares fractions using <, >, = | x | x | x | x | x | | | | |
| **E. Addition Concepts and Skills** | | | | | | | | | |
| 1. Solves addition problems (addition concept, no variables) | x | x | x | x | x | | | | |
| 2. Solves column addition problems (3 addends, with or without regrouping) | x | x | x | x | x | | | | |
| 3. Uses the idea of missing addends (no variables) | x | x | x | x | x | | | | |
| 4. Recites basic addition and multiplication facts (sums to 18) | | | x | x | | | | | |
| 5. Uses the idea of equality and inequalities (without variables): 4 + 1=3 + 2; or 4 • 2=8 • 1 | x | x | x | x | x | | | | |
| 6. Adds money (with regrouping) | x | x | x | x | x | | | | |
| 7. Adds fractions with like and unlike denominators to twelfths | x | | | | x | | | | |
| 8. Adds mixed numbers with like denominators | x | | | | x | | | | |
| **F. Subtraction Concepts and Skills** | | | | | | | | | |
| 1. Solves subtraction problems (subtraction take-away and comparison concepts, differences from 0 - 18) (knowledge) | x | x | x | x | x | | | | |
| 2. Recites basic subtraction facts (differences from 0 - 18) (speed) | | | x | x | | | | | |
| 3. Solves subtraction word problems (requiring no variables, differences from 0 - 18) | x | x | x | x | x | | | | |
| 4. Explains the relationship between addition and subtraction (addition and subtraction are inverse operations) | x | x | x | x | x | | | | |
| 5. Subtracts fractions with like and unlike denominators to twelfths | x | | | | x | | | | |
| 6. Subtracts mixed numbers with like denominators | x | | | | x | | | | |

| Fourth Grade Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| **G. Addition Computation** | | | | | | | | | |
| 1. Adds two- and three-digit numbers of (no regrouping) | x | x | x | | x | | | | |
| 2. Adds two- or three digit-numbers (with regrouping): in ones column, ones and tens columns, all places | x | x | | | x | | | | |
| 3. Adds monetary amounts with and without regrouping | x | | | | x | | | | |
| **H. Subtraction Computation** | | | | | | | | | |
| 1. Subtracts three-digit numbers (no regrouping) | x | x | x | | x | | | | |
| 2. Subtracts three-digit numbers with regrouping in the one, tens, and hundreds | x | x | x | | x | | | | |
| 3. Subtracts three-digit numbers with zeros | x | x | x | | x | | | | |
| 4. Subtracts three-digit numbers (no regrouping) | x | | | | x | | | | |
| 5. Subtracts three- and four-digit numbers with regrouping in the ones and tens places | x | | | | x | | | | |
| 6. Subtracts a one-digit from a two-digit (no regrouping) | x | x | x | | x | | | | |
| 7. Subtracts monetary amounts with and without regrouping | x | | | | x | | | | |
| **I. Multiplication Concepts** | | | | | | | | | |
| 1. Explains and uses the meaning of multiplication (one-digit factors) (knowledge) | x | | | | x | | | | |
| 2. Recites multiplication basic facts (0-81, speed) | | | x | x | | | | | |
| 3. Uses the idea of missing factors (no variables) | x | x | x | x | x | | | | |
| **J. Multiplication Computation** | | | | | | | | | |
| 1. Finds the product of a one-digit number and a multiple of 10 | x | | | | x | | | | |
| 2. Finds the product of a one-digit number and a two-digit number (no regrouping) | x | | | | x | | | | |
| 3. Finds the product of a one-digit number and a two-digit number (with regrouping) | x | | | | x | | | | |
| 4. Finds the product of two-digit number digit numbers (no regrouping) | x | | | | x | | | | |
| 5. Finds the product of a two-digit number and a two-digit number (with regrouping) | x | | | | x | | | | |
| 6. Finds the product of two-digit number digit numbers (with regrouping) | x | | | | x | | | | |
| 7. Multiplies monetary amounts with and without regrouping | x | | | | x | | | | |
| **K. Division Concept** | | | | | | | | | |
| 1. Uses the division concepts (knowledge of sharing and subtractive methods) | x | | | | x | | | | |
| 2. Recites the division basic facts thorough products of 81 (speed) | | | x | x | | | | | |

| Fourth Grade Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| **L. Division Computation** | | | | | | | | | |
| 1. Solves division problems involving single-digit divisors, two-, three- and four-place dividends | x | | | | x | | | | |
| 2. Solves division problems and remainders | x | | | | x | | | | |
| 3. Divides by multiples of 10 | | | x | | x | | | | |
| 4. Finds averages, estimates quotients | x | | | | x | | | | |
| 5. Selects the operation to solve a word problem (no variables) | x | x | | | x | | | | |
| 6. Solves problems using information from the newspaper, catalog ordering, making budgets (no variables) | | | x | | x | | | | |
| **M. Variables, Expressions, Equations, Functions** | | | | | | | | | |
| 1. Uses a literal symbols (x, y, z, a, b, c, or others) to represent (substitute) an unknown quantity (set of whole numbers) | | | x | | x | | | | |
| 3. Uses a literal symbols to represent an unknown in expressions or equations | x | x | x | x | x | | | | |
| 4. Solves addition word problems requiring one variable (expressions or equations) | x | x | x | x | x | | | | |
| 5. Uses the same value for the same variable in equations (sums up to 18): $x + y = 4$, $y = 2$; or $z + 0 = z$ | x | x | x | x | x | | | | |
| 6. Uses the same or different values for different variables in equations (one, two or three variables, sums up to 18): $y + y = 4$ and $y = 2$; $x + y = 3$, $x = 1$ and $y = 2$; or $x + y + z = 3$, x, y and $z = 1$ | x | x | x | x | x | | | | |
| 7. Uses the idea of balancing scales involving addition and missing addends, and one or two variable to represent equations, and equalities | x | x | | | x | | | | |
| 8. Uses the idea of missing addends and factors (one, two or three variables) | x | x | x | x | x | | | | |
| 9. Uses algebraic expressions to translate verbal ideas; for example, "add 5 to another number and the sum is 7" can translated as "$5 + y = 7$", or write an algebraic expression to represent a given situation: weight of a box plus five, and three years less than a certain age as $w + 5$ | x | x | x | | x | | | | |
| 10. Solves addition equations with one or two variables: $3 + 2 = x$, $x = 5$; or $0 + 2 = a$, $a = 2$ | x | x | x | x | x | | | | |
| 11. Uses equations involving addition and multiplication with one or two variables: $3 + 2 = x$, $x = 5$; or $0 + 2 = a$, $a = 2$ | x | | | | x | | | | |

| Fourth Grade Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 12. Uses the idea of equality and inequalities (one, two or three variables): 4 + y = 3 + 2, y = 1; or 4 + x = 3 + y, x = 1 and y = 2 | x | x | x | x | x | | | | |
| 13. Uses the idea of function involving addition and missing addends and one, two or three variables (sums, differences) | x | x | x | x | x | | | | |
| 14. Uses the "ab" to represent "a • b" | | | x | x | | | | | |
| 15. Explores the idea of function multiplication and one, two or three variables (products) | x | x | x | x | x | | | | |
| 16. Finds and extends a pattern (use objects and look for a pattern, no verbal rules) | x | x | | | x | | | | |
| 17. Finds and extends a pattern (use objects and look for a pattern, with verbal rules) | x | x | | | x | | | | |
| 18. Solves subtraction word problems requiring one or two variables (expressions or equations) | x | x | x | x | x | | | | |
| 19. Solves multiplication word problems requiring one, two or three variables (expressions or equations, products) | x | x | x | x | x | | | | |
| 20. Solves division word problems requiring one, or two variables (expressions or equations) | x | x | x | x | x | | | | |
| 21. Explores the formula for averages using variables (generalization) | x | | | | x | | | | |
| 22. Solves problems involving measurements (no variables) | | | | | | | | | |
| 23. Selects the operation to solve a word problem (with variables) | x | x | | | x | | | | |
| 24. Solves problems using information from the newspaper, catalog ordering, making budgets (with variables) | | | x | | x | | | | |
| 25. Solve problems involving measurements (with variables) | | | x | | x | | | | |
| 26. Represents and analyzes patterns and functions, using words, tables, and graphs | x | x | x | x | x | | | | |
| 27. Investigates how a change in one variable relates to a change in a second variable | x | x | | | x | | | | |
| 28. Investigates and describes situations with constant or varying rates of change and compares them | x | x | | | x | | | | |
| **N. Number Properties** | | | | | | | | | |
| 1. Identifies and uses zero as the addition identity element: for all x, x + 0 = x, x = whole number | x | x | x | x | x | | | | |
| 2. Generalizes, identifies and uses the commutative property for addition algebraic expressions: for all a and b, a + b = b + a, a and b = whole numbers | x | x | x | x | x | | | | |

| Fourth Grade Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 3. Generalizes, identifies and uses the associative property for addition using algebraic expressions: for all x, y, and z, x + (y + z) = (x + y) + c, x, y and y represent any whole number | x | x | x | x | x | | | | |
| 4. Generalizes, identifies and uses multiplication commutative property using variables: for any number a and b, a • b = b • a, a and b represent whole numbers | x | x | x | x | x | | | | |
| 5. Generalizes, identifies and uses the associative property for multiplication using algebraic expressions: for all a and b, a • b = b • a, a and b present any whole number | x | x | | | x | | | | |
| 6. Identifies and uses the identity element property for multiplication: for all x, x • 1 = x, x represents any whole number | x | x | x | x | x | | | | |
| 7. Uses the identity element, commutative, and associative properties for addition and multiplication to justify and manipulate equations involving two variables (commutative and identity element for addition), for example, 'Why is x + 4 always equal to 4 + x" | x | x | x | x | x | | | | |
| 8. Explores the distributive property of multiplication over addition (whole numbers): a(b + c) = ab + ac | x | | | | x | | | | |
| **O. Tables and Graphs** | | | | | | | | | |
| 1. Matches a given story to a graph (2 variables) | x | x | | | x | | | | |
| 2. Uses a number line involving whole numbers and fractions | x | x | x | x | x | | | | |
| 3. Graphs data (involving the relationship between two attributes; for example, size and height) | x | x | x | | x | | | | |
| 4. Reads, locates and interprets information from a table: important terms: row, column, top, bottom, first, ordinal numbers | x | x | | | x | | | | |
| 5. Makes a table to organize and solve problems | x | x | | | x | | | | |
| 6. Uses plotting points on x- and y-axes | x | x | | | x | | | | |
| 7. Identifies the generalization for the perimeter formula for any geometric shapes | x | x | | | x | | | | |
| 8. Uses tables to solve problems | | | x | | x | | | | |

Grade 5

| Fifth Grade Algebraic Thinking Checklist | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| **A. Numeration** | | | | | | | | | |
| 1. Counts to 100,000,000,000 | x | x | x | x | x | | | | |
| 2. Compares and orders 2 or more numbers (using =, <, >, whole numbers, integers, decimals) | x | x | x | x | x | | | | |
| 3. Counts using ordinal numbers | | | x | x | x | | | | |
| 4. Uses decimals to thousandths | | | x | x | x | | | | |
| 5. Uses the idea of integers (as an extension of whole numbers, 1-digit numbers, direction and magnitude ideas, number line, two-color chips) | x | x | | | x | | | | |
| 6. Compares integers (using a number line, and two-color chips) | x | x | | | x | | | | |
| 7. Orders integers (using number line and two-color chips | x | x | | | x | | | | |
| **B. Extending Numeration Ideas: Place Value** | | | | | | | | | |
| 1. Uses place value through 999,999,999 | x | x | x | x | x | | | | |
| 2. Uses decimal place value to the hundredths | x | x | x | x | x | | | | |
| 3. Rounds numbers | | | x | x | | | | | |
| 4. Estimates quantities (whole numbers) | x | x | x | x | x | | | | |
| **C. Measurement** | | | | | | | | | |
| 1. Uses metric units to measure (comparing, estimating, measuring): centimeter, meter, liter, milliliter, kilogram, gram, degrees Celsius, changing metric measures 100cm=1 m | x | x | x | x | x | | | | |
| 2. Uses standard units to measure (comparing, estimating, measuring): inch, half-inch, foot, yard, cup, pint, gallon, half-gallon, quart, pound, ounce, changing standard measures | x | x | x | x | x | | | | |
| 3. Uses the ideas of perimeter, area, volume | x | x | x | x | x | | | | |
| 4. Uses the calendar to the week, day, month, year | | | x | x | | | | | |
| 5. Reads 5-, 10-, and 15-minutes multiples after the hour | | | x | x | | | | | |
| 6. Shows time in 5-, 10-, and 15-minutes multiple intervals | x | | | | x | | | | |
| 7. Reads time to the minute | | | x | x | | | | | |
| 8. Shows time to the minute | x | x | x | x | x | | | | |
| 9. Recognizes and show time after the hour, before the hour, fraction portions after and before the hour | x | x | x | x | x | | | | |
| 10. Reads time to the second by using a second hand | | | x | x | | | | | |
| 11. Identifies time zones | | | x | x | | | | | |
| 12. Recognizes penny, nickel, dime, quarter, half-dollars, dollar, other paper currency | x | x | | x | | | | | |
| 13. Counts pennies, nickels, dimes, quarters, half-dollars, dollars | x | x | | | x | | | | |

| **Fifth Grade Algebraic Thinking Checklist Cont. . . .** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 14. Counts mixed type of money in random order | x | x | | | x | | | | |
| 15. Makes purchases > $20.00 | x | | | | x | | | | |
| 16. Makes change to $20.00 | x | | | | x | | | | |
| 17. Identifies numerator and numerator of fractions | | | x | x | | | | | |
| 18. Finds equivalent fractions | x | x | x | x | x | | | | |
| 19. Reads and writes mixed numbers | | | x | x | x | | | | |
| **D. Fractions** | | | | | | | | | |
| 1. Compares fractions using <, >, = | x | x | x | x | x | | | | |
| 2. Reduces or renames fractions to lowest terms | | | x | | x | | | | |
| 3. Explains fractions as ratio | x | x | x | x | x | | | | |
| 4. Uses fractions to represent division | x | x | x | x | x | | | | |
| 5. Represents percents | x | x | x | x | x | | | | |
| 6. Finds percentage | x | x | x | x | x | | | | |
| **E. Addition Concepts and Skills** | | | | | | | | | |
| 1. Solves addition problems (addition concept, no variables) | x | x | x | x | x | | | | |
| 2. Solves column addition problems (3 addends, with or without regrouping) | x | x | x | x | x | | | | |
| 3. Uses the idea of missing addends (no variables) | x | x | x | x | x | | | | |
| 4. Recites basic addition and multiplication facts (sums to 18) | | | x | x | | | | | |
| 5. Uses the idea of equality and inequalities (without variables): 4 + 1 = 3 + 2; or 4 • 2=8 • 1 | x | x | x | x | x | | | | |
| 6. Adds fractions with like and unlike denominators | x | x | | | x | | | | |
| 7. Adds money (with regrouping) | x | x | x | x | x | | | | |
| 8. Adds integers (number line, two-color chips) | x | x | | | x | | | | |
| **F. Subtraction Concepts and Skills** | | | | | | | | | |
| 1. Solves subtraction problems (subtraction take away and comparison concepts, differences from 0 - 18) (knowledge) | x | x | x | x | x | | | | |
| 2. Recites basic subtraction facts (differences from 0 - 18) (speed) | | | x | x | | | | | |
| 3. Solves subtraction word problems (requiring no variables, differences from 0 - 18) | x | x | x | x | x | | | | |
| 4. Explains the relationship between addition and subtraction (addition and subtraction are inverse operations) | x | x | x | x | x | | | | |
| 5. Subtracts fractions with like and unlike denominators | x | x | | | x | | | | |
| 6. Explores the idea subtracting integers (number line, two-color chips) | x | x | | | x | | | | |

| Fifth Grade Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| **G. Addition Computation** | | | | | | | | | |
| 1. Adds large numbers (with and without regrouping in all places): decimals | x | x | | | x | | | | |
| 2. Adds fractions with like and unlike denominators to twelfths | | | x | | x | | | | |
| 3. Adds mixed numbers with like and unlike denominators (with regrouping) | | | x | | x | | | | |
| 4. Adds monetary amounts with and without regrouping | x | | | | x | | | | |
| **H. Subtraction Computation** | | | | | | | | | |
| 1. Subtracts three-digit numbers (no regrouping) | x | x | x | | x | | | | |
| 2. Subtracts with large numbers (with and without regrouping in all places and involving zero subtraction): decimals | x | x | x | | x | | | | |
| 3. Subtracts fractions with like and unlike denominators | | | x | | x | | | | |
| 4. Subtracts mixed numbers with like and unlike denominators (with regrouping) | | | x | | x | | | | |
| 5. Subtracts monetary amounts with and without regrouping | | | x | | x | | | | |
| **I. Multiplication Concepts** | | | | | | | | | |
| 1. Explains and uses the meaning of multiplication (one-digit factors) (knowledge) | x | | | | x | | | | |
| 2. Recites the multiplication basic facts (0-81) (speed) | | | x | x | | | | | |
| 3. Multiplies fractions with like and unlike denominators | x | x | | | x | | | | |
| 4. Uses the idea of missing factors (no variables) | x | x | x | x | x | | | | |
| **J. Multiplication Computation** | | | | | | | | | |
| 1. Finds the product of a one-digit number and a multiple of 10 | x | | | | x | | | | |
| 2. Finds the product of a one-digit number and a two-digit number (no regrouping) | x | | | | x | | | | |
| 3. Finds the product of a one-digit number and a two-digit number (with regrouping) | x | | | | x | | | | |
| 4. Finds the product of two-digit numbers (no regrouping) | x | | | | x | | | | |
| 5. Finds the product of a two-digit number and a two-digit number (with regrouping) | x | | | | x | | | | |
| 6. Finds the product of two-digit numbers (with regrouping) | x | | | | x | | | | |
| 7. Finds the product of three- and four-digit numbers (with and without regrouping) | x | | | | x | | | | |

| Fifth Grade Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 8. Multiplies monetary amounts with and without regrouping | x | | | | x | | | | |
| 9. Multiplies whole numbers and fractions and proper fractions: 3/6 x 9, or 2/3 x 1/5 | | | x | | x | | | | |
| **K. Division Concepts and Skills** | | | | | | | | | |
| 1. Uses the division concepts (knowledge of sharing and subtractive methods) | x | | | | x | | | | |
| 2. Recites the division basic facts thorough products of 81 (speed) | | | x | x | | | | | |
| 3. Explains the relationship between multiplication and division (multiplication and division are inverse operations) | x | x | x | x | x | | | | |
| **L. Division Computation** | | | | | | | | | |
| 1. Solves division problems involving single-digit divisors, two-, three- and four-place dividends | x | | | | x | | | | |
| 2. Solves division problems and with remainders | x | | | | x | | | | |
| 3. Divides by multiples of 10 (with and without zeroes in the quotient) | | | x | | x | | | | |
| 4. Divides into four- and five-place numbers with one- and two -digit divisors | | | x | | x | | | | |
| 5. Estimates quotients involving decimals | | | x | x | x | | | | |
| 6. Divides monetary amounts with and without regrouping: decimals and whole numbers | | | x | | x | | | | |
| 7. Finds averages, estimates quotients | x | | | | x | | | | |
| 8. Selects the operation to solve a word problem (no variables) | x | x | | | x | | | | |
| 9. Identifies prime and composite numbers | x | x | x | x | x | | | | |
| 10. Explains the divisibility rules | | | | | | | | | |
| 11. Solves problems using information from the newspaper, catalog ordering, making budgets (no variables) | | | x | | x | | | | |
| 12. Uses the order of operation rules (no variables) | | | x | | x | | | | |
| 13. Solves problems involving percent, percentage, decimals, fractions and other applications | | | x | | x | | | | |
| **M. Variables, Expressions, Equations, Functions** | | | | | | | | | |
| 1. Uses a literal symbols (x, y, z, a, b, c, or others) to represent (substitute) an unknown quantity (set of whole numbers, decimals, fractions, integers) | | | x | | x | | | | |
| 2. Uses a literal symbols to represent an unknown in expressions or equations | x | x | x | x | x | | | | |
| 3. Solves addition word problems requiring one, two or three variables (expressions or equations) | x | x | x | x | x | | | | |

| Fifth Grade Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 4. Uses the same value for the same variable in equations: x + y = 4, y = 2; or z + 0 = z | x | x | x | x | x | | | | |
| 5. Uses the same or different values for different variables in equations (one, two or three variables, sums up to 18): y + y = 4 and y = 2; x + y = 3, x = 1 and y = 2; or x + y + z = 3, x, y and z = 1 | x | x | x | x | x | | | | |
| 6. Uses the order of operation rules (with variables) | | | x | | x | | | | |
| 7. Uses the idea of balancing scales involving addition, missing addends, multiplication and missing products, and one or two variables to represent equations, equalities and inequalities | x | x | | | x | | | | |
| 8. Uses algebraic expressions to translate verbal ideas, or write an algebraic expression to represent a given situation | x | x | x | | x | | | | |
| 9. Solves addition equations with one, two or three variables: 3+2=x , x=5; or 0+2 = a, a=2 | x | x | x | x | x | | | | |
| 10. Explores equations involving addition and multiplication with one, two or three variables: 3 + 2 = x , x = 5; or 0 + 2 = a, a = 2 | x | x | x | x | x | | | | |
| 11. Uses the idea of equality and inequalities (one, two or three variables): 4 + y = 3 + 2, y = 1; or 4 + x = 3 + y, x = 1 and y = 2 | x | x | x | x | x | | | | |
| 12. Uses the idea of function involving addition, missing addends, multiplication, missing factors, and one, two or three variables | x | x | x | x | x | | | | |
| 13. Use the "ab" to represent "a • b" | | | x | x | | | | | |
| 14. Explores the idea of function for multiplication and one, two or three variables (products) | x | x | x | x | x | | | | |
| 15. Finds and extends the pattern (use objects and look for a pattern, no verbal rules) | x | x | x | x | x | | | | |
| 16. Finds and extends the pattern (use objects and look for a pattern, with verbal rules) | x | x | x | x | x | | | | |
| 17. Solves subtraction word problems requiring one, two or three variables (expressions or equations, differences) | x | x | x | x | x | | | | |
| 18. Solves multiplication word problems requiring one, two or three variables (expressions or equations, products) | x | x | x | x | x | | | | |
| 19. Solves division word problems requiring 1, or 2 variables (expressions or equations, quotients) | x | x | x | x | x | | | | |
| 20. Explores the formula for averages using variables (generalization) | x | | | | x | | | | |

| Fifth Grade Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 21. Solves problems involving measurements (no variables) | | | x | | x | | | | |
| 22. Selects the operation to solve a word problem (with variables) | x | x | | | x | | | | |
| 23. Solves problems using information from the newspaper, catalog ordering, making budgets (with variables) | | | x | | x | | | | |
| 24. Solves problems involving measurements (with variables) | | | x | | x | | | | |
| 25. Identifies and uses zero as the addition identity element: for all x, x + 0 = x, x represents any whole number | x | x | x | x | x | | | | |
| 26. Represents and analyzes patterns and functions, using words, tables, and graphs | x | x | x | x | x | | | | |
| 27. Investigates how a change in one variable relates to a change in a second variable | x | x | x | x | x | | | | |
| 28. Investigates and describes situations with constant or varying rates of change and compares them | x | x | x | x | x | | | | |
| **N. Number Properties** | | | | | | | | | |
| 1. Generalizes, identifies and uses the commutative property for addition using algebraic expressions: for all a and b, a + b = b + a, a and b present any whole number | x | x | x | x | x | | | | |
| 2. Generalizes, identifies and uses the associative property for addition using algebraic expressions: for all x, y, and z, x + (y + z) = (x + y) + c, x, y and y represent any whole number | x | x | x | x | x | | | | |
| 3. Generalizes, identifies and uses multiplication commutative property using variables: for any number a and b, a • b = b • a, a and b represent whole numbers | x | x | x | x | x | | | | |
| 4. Generalizes, identifies and uses the associative property for multiplication using algebraic expressions: for all a and b, a • b = b • a, a and b present any whole number | x | x | x | | x | | | | |
| 5. Identifies and uses the identity element property for multiplication: for all x, x • 1 = x, x represents any whole number | x | x | x | x | x | | | | |
| 6. Uses the identity element, commutative, and associative properties for addition and multiplication to justify and manipulate equations involving 2 variables, for example, 'Why is x + 4 always equal to 4 + x" | x | x | x | x | x | | | | |

| Fifth Grade Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 7. Uses the distributive property of multiplication over addition (whole numbers): a(b+c)=ab+ac | x | x | x | x | x | | | | |
| **O. Tables and Graphs** | | | | | | | | | |
| 1. Matches a given story to a graph (2 variables) | x | x | x | x | x | | | | |
| 2. Uses a number line involving whole numbers and fractions | x | x | x | x | x | | | | |
| 3. Graphs data (involving the relationship between two attributes; for example, size and height) | x | x | x | x | x | | | | |
| 4. Reads, locates and interprets information from a table: important terms: row, column, top, bottom, first, ordinal numbers | x | x | x | x | x | | | | |
| 5. Makes a table to organize and solve problems | x | x | x | x | x | | | | |
| 6. Uses plotting points on x- and y-axes | x | x | x | x | x | | | | |
| 7. Identifies the generalization for the perimeter formula for any geometric shapes | x | x | x | x | x | | | | |
| 8. Uses tables to solve problems | | | x | | x | | | | |

Grades 6-8

| Grades 6-8 Algebraic Thinking Checklist | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| **A. Numeration Ideas: Place Value** | | | | | | | | | |
| 1. Counts to 100,000,000,000,000 | x | x | x | x | x | | | | |
| 2. Compares and orders numbers (using =, <, > symbols, whole numbers, integers, decimals) | x | x | x | x | x | | | | |
| 3. Counts using ordinal numbers | | | x | x | x | | | | |
| 4. Uses decimals to thousandths | | | x | x | x | | | | |
| 5. Uses place value through trillions | x | x | x | x | x | | | | |
| 6. Uses decimal place value to the hundredths | x | x | x | x | x | | | | |
| 7. Rounds numbers | | | x | x | | | | | |
| 8. Estimates quantities | x | x | x | x | x | | | | |
| 9. Expresses numbers in scientific notation | | | x | | x | | | | |
| 10. Uses other number bases | x | x | x | x | x | | | | |
| 11. Uses the idea of integers (as an extension of whole numbers, 1-digit numbers, direction and magnitude ideas, number line, 2-color chips) | x | x | | | x | | | | |
| 12. Compares integers (using a number line, and two-color chips) | x | x | | | x | | | | |
| 13. Orders integers (using number line and two-color chips | x | x | | | x | | | | |
| **B. Measurement** | | | | | | | | | |
| 1. Uses metric units to measure (comparing, estimating, measuring): centimeter, meter, liter, milliliter, kilogram, gram, degrees Celsius, changing metric measures 100cm=1 m | x | x | x | x | x | | | | |
| 2. Uses standard units to measure (comparing, estimating, measuring): inch, half-inch, foot, yard, cup, pint, gallon, half-gallon, quart, pound, ounce, changing standard measures | x | x | x | x | x | | | | |
| 3. Uses the ideas of perimeter, area, volume | x | x | x | x | x | | | | |
| 4. Uses the calendar to the week, day, month, year | | | x | x | | | | | |
| 5. Reads time | | | x | x | | | | | |
| 6. Shows time | x | x | x | x | x | | | | |
| 7. Identifies time zones | | | x | x | | | | | |
| 8. Recognizes penny, nickel, dime, quarter, half-dollars, dollar, other paper currency | x | x | | x | | | | | |
| 9. Counts pennies, nickels, dimes, quarters, half-dollars, dollars | x | x | | | x | | | | |
| 10. Counts mixed type of money in random order | x | x | | | x | | | | |
| 11. Makes purchases > $20.00 | x | | | | x | | | | |
| 12. Makes change > $20.00 | x | | | | x | | | | |
| **C. Fractions** | | | | | | | | | |
| 1. Identifies the numerator and numerator of a fraction | | | x | x | | | | | |
| 2. Finds equivalent fractions | x | x | x | x | x | | | | |

| Grades 6-8 Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 3. Reads and writes mixed numbers | | | x | x | x | | | | |
| 4. Compares fractions using <, >, = | x | x | x | x | x | | | | |
| 5. Reduces or renames fractions to lowest terms | | | x | | x | | | | |
| 6. Explains fractions as ratio | x | x | x | x | x | | | | |
| 7. Uses fractions to represent division | x | x | x | x | x | | | | |
| 8. Represents and finds percents | x | x | x | x | x | | | | |
| 9. Finds percentages | x | x | x | x | x | | | | |
| **D. Addition Concepts and Skills** | | | | | | | | | |
| 1. Solves addition problems (no variables) | x | x | x | x | x | | | | |
| 2. Solves column addition problems (3 addends, with or without regrouping) | x | x | x | x | x | | | | |
| 3. Uses the idea of missing addends (no variables) | x | x | x | x | x | | | | |
| 4. Recites basic addition and multiplication facts (sums to 18) | | | x | x | | | | | |
| 5. Uses the idea of equality and inequality (without variables): 4 + 1=3 + 2; or 4 • 2=8 • 1 | x | x | x | x | x | | | | |
| 6. Adds money (with regrouping) | x | x | x | x | x | | | | |
| 7. Adds fractions with like & unlike denominators | x | | | | x | | | | |
| 8. Adds mixed numbers with like and unlike denominators (with regrouping) | x | x | | | x | | | | |
| 9. Adds integers (number line, two-color chips) | x | x | x | | x | | | | |
| **E. Subtraction Concepts and Skills** | | | | | | | | | |
| 1. Solves subtraction problems (subtraction take-away and comparison concepts (knowledge) | x | x | x | x | x | | | | |
| 2. Recites basic subtraction facts (speed) | | | x | x | | | | | |
| 3. Solves subtraction word problems (requiring no variables) | x | x | x | x | x | | | | |
| 4. Explains the relationship between addition and subtraction | x | x | x | x | x | | | | |
| 5. Subtracts fractions with like and unlike denominators | x | | | | x | | | | |
| 6. Subtracts integers (number line, 2-color chips) | x | x | x | | x | | | | |
| **F. Addition Computation** | | | | | | | | | |
| 1. Adds large numbers (with and without regrouping in all places): decimals | x | x | | | x | | | | |
| 2. Adds fractions with like and unlike denominators, mixed numbers and improper fractions | | | x | | x | | | | |
| 3. Adds monetary amounts with and without regrouping | x | | | | x | | | | |
| **G. Subtraction Computation** | | | | | | | | | |
| 1. Subtracts three-digit numbers (no regrouping) | x | x | x | | x | | | | |

| Grades 6-8 Grades Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 2. Subtracts with large numbers (with and without regrouping in all places and involving zero subtraction): whole numbers and decimals | x | x | x | | x | | | | |
| 3. Subtracts fractions with like and unlike denominators to twelfths | | | x | | x | | | | |
| 4. Subtracts mixed numbers with like and unlike denominators (with regrouping) | | | x | | x | | | | |
| 5. Subtracts monetary amounts with and without regrouping | x | x | x | | x | | | | |
| **H. Multiplication Concepts** | | | | | | | | | |
| 1. Explains and uses the meaning of multiplication (one-digit factors) (knowledge) | x | | | | x | | | | |
| 2. Recites multiplication basic facts (0-81, speed) | | | x | x | | | | | |
| 3. Uses the idea of missing factors (no variables) | x | x | x | x | x | | | | |
| 4. Multiplies fractions: like & unlike denominators | x | x | | | x | | | | |
| 5. Explores the idea of multiplying integers (number line, two-color chips) | x | x | x | | x | | | | |
| **I. Multiplication Computation** | | | | | | | | | |
| 1. Finds the product of a one-digit number and a multiple of 10 or 100 | x | x | x | | x | | | | |
| 2. Finds the product of a one-digit number and a two-digit number (no regrouping) | x | x | x | | x | | | | |
| 3. Finds the product of a one-digit number and a two-digit number (with regrouping) | x | x | x | | x | | | | |
| 4. Finds the product of 2-digit numbers (no regrouping) | x | x | x | | x | | | | |
| 5. Finds the product of a two-digit number and a two-digit number (with regrouping) | x | x | x | | x | | | | |
| 6. Finds the product of two-digit numbers (with regrouping) | | | x | | x | | | | |
| 7. Finds the product of three- and four-digit numbers (with and without regrouping) | | | x | | x | | | | |
| 8. Multiplies monetary amounts with and without regrouping | | | x | | x | | | | |
| 9. Multiplies fractions, mixed number and improper fractions | | | x | | x | | | | |
| **J. Division Concepts and Skills** | | | | | | | | | |
| 1. Uses the division concepts (knowledge of sharing and subtractive methods) | x | | | | x | | | | |
| 2. Recites the division basic facts thorough products of 81 (speed) | | | x | x | | | | | |
| 3. Explains the relationship between multiplication and division (multiplication and division are inverse operations) | x | x | x | x | x | | | | |

| Grades 6-8 Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 4. Divides fractions with like and unlike denominators | x | x | | | x | | | | |
| 5. Explores the idea of dividing integers (number line, two-color chips) | x | x | x | | x | | | | |
| **K. Division Computation** | | | | | | | | | |
| 1. Solves division problems involving single-digit divisors, two- and three- and four-place dividends | x | x | x | | x | | | | |
| 2. Solves division problems with remainders | | | x | | x | | | | |
| 3. Divides by multiples of 10 or 100 (with and without zeroes in the quotient) | | | x | | x | | | | |
| 4. Divides by three -digit divisors | | | x | | x | | | | |
| 5. Divides with decimals in the dividend, the divisor, and in both the dividend and divisor | | | x | | x | | | | |
| 6. Estimates quotients involving decimals | | | x | x | x | | | | |
| 7. Divides monetary amounts with and without regrouping: decimals and whole numbers | | | x | | x | | | | |
| 8. Finds averages, estimates quotients | x | | | | x | | | | |
| 9. Selects the operation to solve a word problem (no variables) | x | x | | | x | | | | |
| 10. Identifies prime and composite numbers | x | x | x | x | x | | | | |
| 11. Explains the divisibility rules | | | | | | | | | |
| 12. Solves problems using information from the newspaper, catalog ordering, making budgets (no variables) | | | x | | x | | | | |
| 13. Divides fractions and whole numbers | | | x | | x | | | | |
| 14. Divides mixed numbers | | | x | | x | | | | |
| 15. Solves problems involving percent, percentage, decimals, fractions and other applications | | | x | | x | | | | |
| **L. Variables, Expressions, Equations, Functions** | | | | | | | | | |
| 1. Uses a literal symbols (x, y, z, a, b, c, or others) to represent (substitute) an unknown quantity (set of whole numbers, decimals, fractions, integers) | | | x | | x | | | | |
| 2. Uses a literal symbols to represent an unknown in expressions or equations | x | x | x | x | x | | | | |
| 3. Solves addition word problems requiring 1, 2 or 3 variables (expressions or equations) | x | x | x | x | x | | | | |
| 4. Uses the same value for the same variable in equations: $x + y = 4$, $y = 2$; or $z + 0 = z$ | x | x | x | x | x | | | | |
| 5. Uses the same or different values for different variables in equations (1, 2 or 3 variables | x | x | x | x | x | | | | |
| 6. Balances scales involving +, missing addends, x, missing products, and 1 or 2 variables to represent equations, equalities and inequalities | x | x | | | x | | | | |

| Grades 6-8 Grades Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 7. Uses algebraic expressions to translate verbal ideas, or write an algebraic expression to represent a given situation | x | x | x | | x | | | | |
| 8. Names the independent and dependent variable in a problem | x | x | x | x | x | | | | |
| 9. Recognizes what varies in an experiment | x | x | x | x | x | | | | |
| 10. Recognizes rate of change in a pattern (two variables) | x | x | x | x | x | | | | |
| 11. Recognizes the relationship among a table, a graph, and symbolic expression | | | x | | x | | | | |
| 12. Identifies the existence of linearity in a given situation | x | x | x | x | x | | | | |
| 13. Identifies the y-intercept from a graph or a table | | x | x | x | x | | | | |
| 14. Explores the relationship among line, slopes, and y-intercepts | | | x | | x | | | | |
| 15. Compares and contrasts linear relationships with other kinds of relationships | | | | | | | | | |
| 16. Solves addition equations with one, two or three variables: 3 + 2 = x , x=5; or 0+2=a, a=2 | x | x | x | x | x | | | | |
| 17. Explores equations involving addition and multiplication with one, two or three variables | x | x | x | x | x | | | | |
| 18. Uses the idea of equality and inequalities (one, two or three variables): 4 + y = 3 + 2, y = 1; or 4 + x = 3 + y, x = 1 and y = 2 | x | x | x | x | x | | | | |
| 19. Uses the idea of function involving addition, missing addends, multiplication, missing f actors, and one, two or three variables | x | x | x | x | x | | | | |
| 20. Uses the "ab" to represent "a • b" | | | x | x | | | | | |
| 21. Explores the idea of function for multiplication and one, two or three variables (products) | x | x | x | x | x | | | | |
| 22. Finds and extends the pattern (use objects and look for a pattern, no verbal rules) | x | x | x | x | x | | | | |
| 23. Finds and extends the pattern (use objects and look for a pattern, with verbal rules) | x | x | x | x | x | | | | |
| 24. Represents, analyzes, and generalizes a variety of patterns with tables, graphs, words, and when possible, symbolic rules | x | x | x | x | x | | | | |
| 25. Relates and compares different forms of representations for a relationship | x | x | x | x | x | | | | |
| 26. Identifies functions as linear or nonlinear and contrast their properties form tables, graphs, or equations | | | x | x | | | | | |

| Grades 6-8 Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 27. Explores relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope | | | x | | x | | | | |
| 28. Solves subtraction word problems requiring 1, 2 or 3 variables (expressions or equations) | x | x | x | x | x | | | | |
| 29. Solves multiplication word problems requiring one, two or three variables (expressions or equations, products) | x | x | x | x | x | | | | |
| 30. Solves division word problems requiring 1, or 2 variables (expressions or equations) | x | x | x | x | x | | | | |
| 31. Explores the formula for averages using variables (generalization) | x | | | | x | | | | |
| 32. Uses symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships | | | x | | x | | | | |
| 33. Recognizes and generates equivalent forms for imple algebraic expressions and solve linear equations | | | x | | x | | | | |
| 34. Models and solves contextual problems using various representations, such as graphs, tables, and equations | | | | | | | | | |
| 35. Uses graphs to analyze the nature of changes in quantities in linear relationships | | x | x | | x | | | | |
| **M. Number Properties** | | | | | | | | | |
| 1. Solves problems involving measurements (no variables) | | | x | | x | | | | |
| 2. Selects the operation to solve a word problem (with variables) | x | x | | | x | | | | |
| 3. Solves problems using information from the newspaper, catalog ordering, making budgets (with variables) | | | x | | x | | | | |
| 4. Solves problems involving measurements (with variables) | | | x | | x | | | | |
| 5. Identifies and uses zero as the addition identity element: for all x, $x + 0 = x$, x represents any whole number | x | x | x | x | x | | | | |
| 6. Generalizes, identifies and uses the commutative property for addition using algebraic expressions | x | x | x | x | x | | | | |
| 7. Generalizes, identifies and uses the associative property for addition using algebraic expressions: for all x, y, and z, $x + (y + z) = (x + y) + c$, x, y and y represent any whole number | x | x | x | x | x | | | | |

| Grades 6-8 Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 8. Generalizes, identifies and uses multiplication commutative property using variables: for any number a and b, a • b = b • a, a and b represent whole numbers | x | x | x | x | x | | | | |
| 9. Generalizes, identifies and uses the associative property for multiplication using algebraic expressions: for all a and b, a • b = b • a, a and b present any whole number | x | x | x | | x | | | | |
| 10. Identifies and uses the identity element property for multiplication: for all x, x • 1 = x, x represents any whole number | x | x | x | x | x | | | | |
| 11. Uses the identity element, commutative, and associative properties for addition and multiplication to justify and manipulate equations involving two variables, for example, 'Why is x + 4 always equal to 4 + x" | x | x | x | x | x | | | | |
| 12. Uses the distributive property of multiplication over addition (whole numbers): a(b + c) = ab + ac | x | x | x | x | x | | | | |
| **N. Tables and Graphs** | | | | | | | | | |
| 1. Matches a given story to a graph (two variables) | x | x | x | x | x | | | | |
| 2. Uses a number line involving whole numbers and fractions | x | x | x | x | x | | | | |
| 3. Graphs data (involving the relationship between two attributes; for example, size and height) | x | x | x | x | x | | | | |
| 4. Reads, locates and interprets information from a table | x | x | x | x | x | | | | |
| 5. Makes a table to organize and solve problems | x | x | x | x | x | | | | |
| 6. Uses plotting points on x- and y-axes | x | x | x | x | x | | | | |
| 7. Identifies the generalization for the perimeter formula for any geometric shapes | x | x | x | x | x | | | | |
| 8. Uses tables to solve problems | | | x | | x | | | | |

Grades 9-12

| Grades 9-12 Algebraic Thinking Checklist | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| **A. Numeration Ideas: Place Value** | | | | | | | | | |
| 1. Counts to 100,000,000,000,000 | x | x | x | x | x | | | | |
| 2. Compares and orders numbers (using =, <, > symbols, whole numbers, integers, decimals) | x | x | x | x | x | | | | |
| 3. Counts using ordinal numbers | | | x | x | x | | | | |
| 4. Uses decimals to thousandths | | | x | x | x | | | | |
| 5. Uses place value through trillions | x | x | x | x | x | | | | |
| 6. Uses decimal place value to the hundredths | x | x | x | x | x | | | | |
| 7. Rounds numbers | | | x | x | | | | | |
| 8. Estimates quantities | x | x | x | x | x | | | | |
| 9. Expresses numbers in scientific notation | | | x | | x | | | | |
| 10. Uses other number bases | x | x | x | x | x | | | | |
| 11. Uses the idea of integers (as an extension of whole numbers, one-digit numbers, direction & magnitude ideas, number line, 2-color chips) | x | x | x | x | x | | | | |
| 12. Compares integers (using a number line, and two-color chips) | x | x | x | x | x | | | | |
| 13. Orders integers (using number line and two-color chips | x | x | x | x | x | | | | |
| **B. Measurement** | | | | | | | | | |
| 1. Uses metric units to measure (comparing, estimating, measuring): centimeter, meter, liter, milliliter, kilogram, gram, degrees Celsius, changing metric measures 100cm=1 m | x | x | x | x | x | | | | |
| 2. Uses standard units to measure (comparing, estimating, measuring): inch, half-inch, foot, yard, cup, pint, gallon, half-gallon, quart, pound, ounce, changing standard measures | x | x | x | x | x | | | | |
| 3. Uses the ideas of perimeter, area, volume | x | x | x | x | x | | | | |
| 4. Uses the calendar to the week, day, month, year | | | x | x | | | | | |
| 5. Reads time | | | x | x | | | | | |
| 6. Shows time | x | x | x | x | x | | | | |
| 7. Identifies time zones | | | x | x | | | | | |
| 8. Counts mixed type of money in random order | x | x | | | x | | | | |
| 9. Makes purchases > $20.00 | x | | | | x | | | | |
| 11. Makes change > $20.00 | x | | | | x | | | | |
| **C. Fractions** | | | | | | | | | |
| 1. Identifies numerator and numerator fractions | | | x | x | | | | | |
| 2. Finds equivalent fractions | x | x | x | x | x | | | | |
| 3. Reads and writes mixed numbers | | | x | x | x | | | | |
| 4. Compares fractions using <, >, = | x | x | x | x | x | | | | |
| 5. Reduces or renames fractions to lowest terms | | | x | | x | | | | |

| Grades 9-12 Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 6. Explains fractions as ratio | x | x | x | x | x | | | | |
| 7. Uses fractions to represent division | x | x | x | x | x | | | | |
| 8. Represents and finds percents | x | x | x | x | x | | | | |
| 9. Finds percentage | x | x | x | x | x | | | | |
| 10. Finds and solves proportions | x | x | x | x | x | | | | |
| **D. Addition Concepts and Skills** | | | | | | | | | |
| 1. Solves addition problems (no variables) | x | x | x | x | x | | | | |
| 2. Solves column addition problems (3 addends, with or without regrouping) | x | x | x | x | x | | | | |
| 3. Uses the idea of missing addends (no variables) | x | x | x | x | x | | | | |
| 4. Recites basic addition and multiplication facts (sums to 18) | | | x | x | | | | | |
| 5. Uses the idea of equality and inequality (without variables): 4 + 1=3 + 2; or 4 • 2=8 • 1 | x | x | x | x | x | | | | |
| 6. Adds money (with regrouping) | x | x | x | x | x | | | | |
| 7. Adds fractions with like & unlike denominators | x | | | | x | | | | |
| 8. Adds mixed numbers with like and unlike denominators (with regrouping) | x | x | | | x | | | | |
| 9. Adds integers (number line, two-color chips) | x | x | x | x | x | | | | |
| **E. Subtraction Concepts and Skills** | | | | | | | | | |
| 1. Solves subtraction problems (subtraction take-away and comparison concepts (knowledge) | x | x | x | x | x | | | | |
| 2. Recites basic subtraction facts (speed) | | | x | x | | | | | |
| 3. Solves subtraction word problems (requiring no variables) | x | x | x | x | x | | | | |
| 4. Explains the relationship between addition and subtraction (addition and subtraction are inverse operations) | x | x | x | x | x | | | | |
| 5. Subtracts fractions with like and unlike denominators | x | | | | x | | | | |
| 6. Subtracts integers (number line, 2-color chips) | x | x | x | x | x | | | | |
| **F. Addition Computation** | | | | | | | | | |
| 1. Adds large numbers (with and without regrouping in all places): decimals | x | x | | | x | | | | |
| 2. Adds fractions with like and unlike denominators, mixed numbers and improper fractions | | | x | | x | | | | |
| 3. Adds monetary amounts with and without regrouping | x | | | | x | | | | |
| **G. Subtraction Computation** | | | | | | | | | |
| 1. Subtracts three-digit numbers (no regrouping) | x | x | x | | x | | | | |
| 2. Subtracts with large numbers (with and without regrouping in all places and involving zero subtraction): decimals | x | x | x | | x | | | | |

| Grades 9-12 Grades Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 3. Subtracts fractions with like and unlike denominators to twelfths | | | x | | x | | | | |
| 4. Subtracts mixed numbers with like and unlike denominators (with regrouping) | | | x | | x | | | | |
| 5. Subtracts monetary amounts with and without regrouping | x | x | x | | x | | | | |
| **H. Multiplication Concepts** | | | | | | | | | |
| 1. Explains and uses the meaning of multiplication (one-digit factors) (knowledge) | x | | | | x | | | | |
| 2. Recites multiplication basic facts (0-81) (speed) | | | x | x | | | | | |
| 3. Uses the idea of missing factors (no variables) | x | x | x | x | x | | | | |
| 4. Multiplies fractions: like and unlike denominators | x | x | | | x | | | | |
| 5. Multiplies integers (number line, 2-color chips) | x | x | x | | x | | | | |
| **I. Multiplication Computation** | | | | | | | | | |
| 1. Finds products of a 1-digit number and multiples of 10 or 100 | x | x | x | | x | | | | |
| 2. Finds the product of a one-digit number and a two-digit number (no regrouping) | x | x | x | | x | | | | |
| 3. Finds the product of a one-digit number and a two-digit number (with regrouping) | x | x | x | | x | | | | |
| 4. Finds products of 2-digit numbers (no regrouping) | x | x | x | | x | | | | |
| 5. Finds the product of a two-digit number and a two-digit number (with regrouping) | x | x | x | | x | | | | |
| 6. Finds the product of two-digit numbers (with regrouping) | | | x | | x | | | | |
| 7. Finds the product of three- and four-digit numbers (with and without regrouping) | | | x | | x | | | | |
| 8. Multiplies monetary amounts with and without regrouping | | | x | | x | | | | |
| 9. Multiplies fractions, mixed number and improper fractions | | | x | | x | | | | |
| **K. Division Concepts and Skills** | | | | | | | | | |
| 1. Uses the division concepts (knowledge of sharing and subtractive methods) | x | | | | x | | | | |
| 2. Recites the division basic facts thorough products of 81 (speed) | | | x | x | | | | | |
| 3. Explains the relationship between multiplication and division (multiplication and division are inverse operations) | x | x | x | x | x | | | | |
| 4. Divides fractions with like and unlike denominators | x | x | | | x | | | | |
| 5. Divides integers (number line, two-color chips) | x | x | x | x | x | | | | |

| Grades 9-12 Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| **L. Division Computation** | | | | | | | | | |
| 1. Solves division problems involving single-digit divisors, 2- and 3- and 4-place dividends | x | x | x | | x | | | | |
| 2. Solve division problems with remainders | | | x | | x | | | | |
| 3. Divides by multiples of 10 or 100 (with and without zeroes in the quotient) | | | x | | x | | | | |
| 4. Divides by three -digit divisors | | | x | | x | | | | |
| 5. Divides with decimals in the dividend, the divisor, and in both the dividend and divisor | | | x | | x | | | | |
| 6. Estimates quotients involving decimals | | | x | x | x | | | | |
| 7. Divides monetary amounts with and without regrouping: decimals and whole numbers | | | x | | x | | | | |
| 8. Finds averages, estimates quotients | x | | | | x | | | | |
| 9. Selects the operation to solve a word problem (no variables) | x | x | | | x | | | | |
| 10. Identifies prime and composite numbers | x | x | x | x | x | | | | |
| 11. Explains the divisibility rules | | | | | | | | | |
| 12. Solves problems using information from the newspaper, catalog ordering, making budgets (no variables) | | | x | | x | | | | |
| 13. Divides fractions and whole numbers | | | x | | x | | | | |
| 14. Divides mixed numbers | | | x | | x | | | | |
| 15. Solves problems involving percent, percentage, decimals, fractions and other applications | | | x | | x | | | | |
| **M. Variables, Expressions, Equations, Functions** | | | | | | | | | |
| 1. Uses a literal symbols (x, y, z, a, b, c, or others) to represent (substitute) an unknown quantity (set of whole numbers, decimals, fractions, integers) | | | x | | x | | | | |
| 2. Uses a literal symbols to represent an unknown in expressions or equations | x | x | x | x | x | | | | |
| 3. Solves addition word problems requiring 1, 2 or 3 variables (expressions or equations) | x | x | x | x | x | | | | |
| 4. Uses the same value for the same variable in equations: $x + y = 4$, $y = 2$; or $z + 0 = z$ | x | x | x | x | x | | | | |
| 5. Uses the same or different values for different variables in equations (1, t2 or 3 variables: $y + y = 4$ and $y = 2$; $x + y = 3$, $x = 1$ and $y = 2$; or $x + y + z = 3$, x, y and $z = 1$ | x | x | x | x | x | | | | |
| 6. Names the independent and dependent variable in a problem | x | x | x | x | x | | | | |
| 7. Recognizes what varies in an experiment | x | x | x | x | x | | | | |
| 8. Recognizes rate of change in a pattern (two variables) | x | x | x | x | x | | | | |

| Grades 9-12 Grades Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 9. Recognizes the relationship among a table, a graph, and symbolic expression | | | x | | x | | | | |
| 10. Identifies the existence of linearity in a given situation | x | x | x | x | x | | | | |
| 11. Identifies the y-intercept from a graph or a table | | x | x | x | x | | | | |
| 12. Explores the relationship among lines, slopes, and y-intercepts | | | x | | x | | | | |
| 13. Compares and contrasts linear relationships with other kinds of relationships | | | | | | | | | |
| 14. Uses and solves equations involving addition and multiplication with 1, 2 or 3 variables | x | x | x | x | x | | | | |
| 15. Uses the idea of equality and inequalities (one, two or three variables) | x | x | x | x | x | | | | |
| 16. Uses the "ab" to represent "a • b" | | | x | x | | | | | |
| 17. Uses the idea of function for multiplication and one, two or three variables (products) | x | x | x | x | x | | | | |
| 18. Finds and extends the pattern (use objects and look for a pattern, no verbal rules) | x | x | x | x | x | | | | |
| 19. Finds and extends the pattern (use objects and look for a pattern, with verbal rules) | x | x | x | x | x | | | | |
| 20. Solves subtraction word problems requiring one, two or three variables (expressions or equations, differences) | x | x | x | x | x | | | | |
| 21. Solves multiplication word problems requiring one, two or three variables (expressions or equations, products) | x | x | x | x | x | | | | |
| 22. Solves division word problems requiring 1, or 2 variables (expressions or equations, quotients) | x | x | x | x | x | | | | |
| 23. Uses the formula for averages using variables (generalization) | x | | | | x | | | | |
| 24. Solves problems involving measurements (no variables) | | | x | | x | | | | |
| 25. Selects the operation to solve a word problem (with variables) | x | x | | | x | | | | |
| 26. Solves problems using information from the newspaper, catalog ordering, making budgets (with variables) | | | x | | x | | | | |
| 27. Solves problems involving measurements (with variables) | | | x | | x | | | | |
| 28. Explains the meaning of equivalent forms of expressions, equations, inequalities, and relations | x | x | x | x | x | | | | |
| 29. Writes equivalent forms of equations, inequalities, and systems of equations | | | x | | x | | | | |

| Grades 9-12 Grades Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 30. Solves equivalent forms of equations, inequalities, and systems of equations with fluency (uses appropriate methods: mentally, with paper and pencil in simple cases, and using technology in all cases) | | | x | | x | | | | |
| 31. Uses algebraic expressions to translate verbal ideas, or write an algebraic expression to represent a given situation and to explain mathematical relationships | x | x | x | | x | | | | |
| 32. Uses a variety of symbolic representations, including recursive and parametric equations, for functions and relations | | | x | | x | | | | |
| 33. Judges the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out with technology | | | x | | x | | | | |
| 34. Explains the idea of function | x | x | x | x | x | | | | |
| 35. Generalizes patterns using explicitly and recursively defined functions | x | x | x | x | x | | | | |
| 36. Analyzes and uses functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behaviors | | | x | | x | | | | |
| 37. Selects, converts flexibly among and uses various representations for relations and functions | x | x | x | x | x | | | | |
| 38. Explains and performs transformations: arithmetically combining such as composing and inverting commonly used functions, using technology to perform such operations on more complicated symbolic expressions | | | x | | x | | | | |
| 39. Explains and compares the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions | | | x | | x | | | | |
| 40. Interprets representations of functions of two variables | | | x | | x | | | | |
| 41. Identifies essential quantitative relationships in a situation and determines the class or classes of functions that might model the relationships | | | x | | x | | | | |
| 42. Uses symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts | | | x | | x | | | | |
| 43. Draws reasonable conclusions about a situation being modeled | | | x | | x | | | | |
| 44. Approximates and interprets rates of change from graphical and numerical data | | | x | | x | | | | |

| Grades 9-12 Algebraic Thinking Checklist Cont. . . . | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Skill/Concept** | **Representation** | | | **Validation** | | **Mastery** | | **Retention** | |
| | Concrete | Pictorial | Abstract | Say | Do | Yes | No | Yes | No |
| 45. Recognizes the properties of families of functions | | | x | | x | | | | |
| 46. Applies properties of functions | | | x | | x | | | | |
| 47. Recognizes what conditions are necessary to divide functions | | | x | | x | | | | |
| 48. Operates on functions | | | x | | x | | | | |
| **N. Number Properties** | | | | | | | | | |
| 1. Generalizes, identifies and uses addition and multiplication commutative property using variables: for any number a and b, a • b = b • a, a and b represent whole numbers | x | x | x | x | x | | | | |
| 2. Generalizes, identifies and uses the associative property for addition and multiplication using algebraic expressions | x | x | x | x | x | | | | |
| 3. Identifies and uses the identity element for addition and multiplication | x | x | x | x | x | | | | |
| 4. Uses the distributive property of multiplication over addition: a(b + c) = ab + ac | x | x | x | x | x | | | | |
| 5. Uses properties to justify and manipulate equations involving 1, 2 or 3 variables | x | x | x | x | x | | | | |
| **O. Tables and Graphs** | | | | | | | | | |
| 1. Matches a given story to a graph (2 variables) | x | x | x | x | x | | | | |
| 2. Uses a number line involving whole numbers and fractions | x | x | x | x | x | | | | |
| 3. Graphs data (involving the relationship between two attributes; for example, size and height) | x | x | x | x | x | | | | |
| 4. Reads, locates and interprets information from a table | x | x | x | x | x | | | | |
| 5. Makes a table to organize and solve problems | x | x | x | x | x | | | | |
| 6. Uses plotting points on x- and y-axes | x | x | x | x | x | | | | |
| 7. Identifies the generalization for the perimeter formula for any geometric shapes | x | x | x | x | x | | | | |
| 8. Uses tables to solve problems | | | x | | x | | | | |

**Reference**

National Council of Teacher of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, Va: NCTM, 2001.