Tab 1, Handout 7
Research Findings from NRC, 2001 Report

Finding #1:

Knowledge learned with understanding provides a foundation for generating new knowledge and for solving unfamiliar problems. (Bransford et al., 1999) For example, students who understand place value and other multidigit number concepts are more likely than students without such understanding to invent their own procedures for multicolumn addition and to adopt correct procedures for multicolumn subtraction that others have presented to them. (Hiebert and Wearne, 1996)

Finding #2:

In the brain, the way that new knowledge is organized and connected to previous knowledge is critical to the ability to retrieve and apply that knowledge. Learning with understanding leads to better organization and connections in the brain than does memorizing. (Donovan et al., 1999) When students learn without understanding they learn isolated bits of knowledge. Learning new topics is then more difficult because there is no network of previously learned concepts to link a new topic to. (Saxe, 1990)

Finding #3:

A good conceptual understanding of place value in the base-ten system supports multidigit computational fluency, accurate mental arithmetic, and flexibility with numbers. (several references cited here, including Fuson, 1990)

Finding #4:

Justifying and explaining ideas improves students' reasoning skills and their conceptual understanding. (Maher and Martino, 1996)

Finding #5:

Teachers need to expand the study of data beyond just graphing data. Four key processes are describing, organizing, representing, and analyzing data. (Shaughnessy et al., 1996)

Finding #6:

Elementary students are capable of learning more geometry than is usually taught. Given enough early opportunities to learn about geometric figures, by the end of second grade they should be able to identify a wide range of examples and nonexamples of geometric figures; classify, describe, draw, and visualize shapes; and describe and compare shapes based on their attributes. (Clements, 2000)

Finding #7:

Students emerging from traditional elementary school arithmetic have developed habits that make the study of algebra more difficult. For example, they have an orientation to execute operations rather than to use them to represent relationships, which leads to the use of the equal sign to announce a result rather than to signify an equality. They also have trouble moving from an addition statement written horizontally to its equivalent subtraction statement (e.g., writing $35 + 42 = 77$ as $35 = 77 - 42$, or writing $x + 42 = 77$ as $x = 77 - 42$). (Thompson et al., 1994)

Finding #8:

The most effective sets of activities for teaching about rational numbers spend time at the outset helping students develop meaning for the different forms of representation. Students work with multiple physical models for rational numbers as well as pictures, realistic contexts, and verbal descriptions. Time is spent helping students connect these supports with the written symbols for rational numbers. (several references, including Cramer et al., in press)

Finding #9:

How a teacher views mathematics and its learning affects his/her teaching practice, which in turn affects what students learn and how they view themselves as mathematics learners. (Thompson, 1992)

Finding #10:

When class norms allow for students to feel comfortable doing mathematics and sharing their ideas with others, students see themselves as capable of understanding. (Cobb et al., 1995)

If these findings intrigue you, explore the report online. In particular, I recommend Chapter 4: "The Strands of Mathematical Proficiency" (31 pages without the references). The definition of mathematical proficiency as five intertwined strands serves as a framework for the rest of this insightful report.

Wendy Gulley, CESAME, Northeastern University
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