Subject Area: Mathematics

Domain: Statistics & Probability

Cluster: Investigate chance processes and develop, use, and evaluate probability models.

Desired Results

Identify desired results

What relevant goals (standards, course or program objectives, learning outcomes) will this design address?

Standard

MAFS.7.SP.3.7: Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.
b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

Cognitive Complexity: Level 3, Strategic Thinking and Complex Reasoning

Access Points

MAFS.7.SP.3.AP.7a: Compare actual results of a simple experiment when numbers of instances are increased.

Aligned Prior Knowledge

MAFS.7.SP.3.5: Understand that the probability of a chance event is a number between 0 and 1, and expresses the likelihood of an event occurring. Larger numbers indicate a greater likelihood and smaller numbers indicate an unlikely event.

MAFS.7.SP.3.6: Approximate the probability of a chance event by collecting data on the process and observing it's long-run relative frequency, and predict the approximate relative frequency given the probability.

Aligned Subsequent Knowledge

MAFS.7.SP.3.8: Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.
c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

Supporting Access Points

MAFS.7.SP.3.AP.5a: Define the probability of related events given a situation of chance.

MAFS.7.SP.3.AP.6a: Make a prediction regarding the probability of an event occurring; conduct simple probability experiments and compare results to predictions.

Supporting Access Points

MAFS.7.SP.3.AP.8a: Determine the theoretical probability of compound events (e.g., two coins or two dice).

MAFS.7.SP.3.AP.8b: Use tree diagrams, frequency tables, organized lists, and/or simulations to collect data from a two-step simulation of compound events (using two coins and/or two dice).

Deconstruct Standard Concepts
Students will know . . .

- What key knowledge and skills will students acquire as a result of this work?
- Underline Nouns/Phrases
    - Probability model
    - Probability of events

Deconstruct Standards Skills
Students will be able to . . .

- What should they eventually be able to do as a result of such knowledge and skills?
- Circle Verbs/Verb Phrases
    - Develop
    - Use

• Observed frequencies • Possible sources (of the discrepancy) • Outcomes • Frequencies in data (generated from a chance process)	• Compare • Explain • Assigning • Observing

Explicit Prerequisite Knowledge
 	
• What skills should students be proficient in prior to this standard?
 	
  • Understand how to determine the probability of an event by examining the possible equally-likely outcomes.
  • Understand the relationship between a probability and a ratio.
  • Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring.
  • Part to whole relationships
  • How to change a fraction to a decimal
  • How to change a decimal to a percent
	Implicit Skills
	
	• What skills are necessary for students to access and engage with instruction and demonstrate their learning?
	
	  • Access grade level texts, topics and issues
	  • Set goals, plan, and monitor progress
	  • Utilize frequent opportunities to collaborate and team with peers
	  • Exhibit active listening and communication skills
	  • Practice perspective taking and an appreciation of diversity
	  • Engage in tasks and assignments that allow for multiple and creative approaches
	  • Pose and respond to questions

Learning Goals:
 	
A learning goal is a statement of what learners will know and/or be able to do.
• "I can" statement, in student friendly language
 	
  • I can understand that the probability of a compound event is the product of the probabilities of the individual events.
  • I can calculate the theoretical probability of a compound event.
  • I can calculate the experimental probability of a compound event.
  • I can create a tree diagram representing the sample space of a compound event.
  • I can compare the theoretical and experimental probability of a compound event using gathered data and sample space.
  • I can explain their own reasoning and critique the reasoning of others.
	Essential Question(s):
	
	What questions will foster inquiry, understanding, and transfer of learning?
	
	  • Does the probability of an event change or remain the same, if you add multiple events or trials? For example; if you toss a coin twice and would like to find the probability of the coin landing on heads for each toss, is the probability still ½ or 50 %?
	  • How do we represent the sample space for a compound event?
	  • What is the difference between dependent and independent events?
	  • How do we simulate events to determine experimental probability?

High Probability Barrier(s) Wide-spread or common barriers that impact many students' engagement and learning (e.g., integrate strategies that support cognitive processing through academic instruction, DI, provide adequate instructional time)	**High Intensity Barrier(s)** Significant impact on individual student engagement and learning (e.g., small group & individual instruction, Differentiated Instruction (DI), aligned with learning needs)
Instruction: • Time with instruction to address lack of background knowledge • Teacher provides one way to learn the material (verbally) • Access to grade level texts • Texts not aligned to standards • Classroom management	**Instruction:** • Unsure of strategies for teaching ELL/ESE (most struggling learners) • Opportunities to collaborate and team with peers
Curriculum: • Limited resources/technology tools • Worksheets	**Curriculum:** • Only provided grade level material (lack of supplemental)

	• Monolingual (only English) • Limited resources/technology tools
Environment: • Classroom setup limits student interaction • Distracting learning environment	**Environment:** • Classroom setup limits student interaction • Distracting learning environment
Learner: • Lack of academic and domain-specific vocabulary • Setting goals, planning, monitoring progress • Engagement • Information not interesting or relevant • Ability to use math tools appropriately • Ability to calculate precisely	**Learner:** • Below grade level reading skills • Lack of engagement • ELL • Setting goals, planning, monitoring progress • Exhibit active listening and communication skills • Pose and respond to questions • Physical disabilities • Visual/hearing impairments • Behavioral issues • Ability to use math tools appropriately • Ability to calculate precisely

Instructional Design

Depending upon the anticipated barriers above, what implications would these have on the design of your lesson(s) regarding this standard.

The term "tiers" is often used to communicate a hierarchical relationship among elements in a complex system. For example, the broad instructional design of a multi-tiered system of supports (MTSS) in Florida addresses three tiers or levels of academic and behavioral support aligned to ongoing formative and interim assessments of student learning needs. Tier 1 instruction & support is provided to all students and includes differentiation to meet a variety of needs. Tier 2 is supplemental instruction and supports provided in addition to and integrated with Tier 1 instruction to smaller groups of students who demonstrate need for that level of instruction. Tier 3 is the most intensive and individualized level of instruction in addition to and integrated with Tier 1 for specific students based on unique needs. At the core of implementing a MTSS framework is the systematic use of a data-based problem solving and decision making process that must be integrated seamlessly into all systems planning.

Tier 1 Core Universal Instruction and Supports – General academic and behavior instruction and supports, based on Universal Design for Learning principles, designed and differentiated for all students in all settings.	**Tier 2** Targeted Supplemental Interventions and Supports – More focused, targeted instruction/intervention and supplemental supports in addition to and aligned with the core academic and behavior curriculum and instruction. Evidence-based instruction provided in the general education setting.	**Tier 3** Intensive Individualized Interventions and Supports – The most intense (increased time, narrowed focus, reduced group size) instruction and intervention based upon individual student need provided in addition to and aligned with core and supplemental academic and behavior, curriculum, instruction, and supports. Evidence-based instruction provided in the general education setting.
• Number Talks • Determine and address background knowledge that is missing and is foundational for the focus of this standard • Provide student choice for task • Provide note-taking tools (e.g., graphic organizers, highlighters) • Research resources (e.g., CPALMS, additional curriculum materials, district resource map) • Learning spaces are designed to support instructional activities • Link new vocabulary to prior knowledge • Include students in setting goals,	• Research resources (e.g., CPALMS, supplemental curriculum materials, district resource map) • Pre-teach vocabulary • Target skill gaps within small groups using data informed instruction • Peer coaching	• Determine skill gaps to address foundational deficits (i.e., number sense) • Pre-teach vocabulary • Reduce/highlight text to focus students

planning, and monitoring progress (e.g., rubrics) • Implement engagement practices (e.g., flexible grouping) • Provide flexible, digital text that can be modified by the widest range of students • Address misconceptions		

Specially Designed Instruction

Specially Designed Instruction is a service, not a place, and is not defined by where it occurs and must be provided in the least restrictive environment.

- Refer to IEP goals and accommodations
    - Physical or cognitive disabilities (e.g., unable to write answers to problems or needs alternative to demonstrating learning through using a computer app or verbally explaining how problem was solved) accommodations provided in general education classroom, allowing access to Tier 1 instruction.

Assessment Evidence

Determine acceptable evidence

Performance Tasks (in relation to the standard):

- Through what authentic performance tasks will students demonstrate growth toward proficiency of the standard?
- Are the performance indicators (i.e., outcomes) based on process, effort, and improvement?
    - Interactions allowing students to express the probability of an independent event as a fraction, decimal, and percent by collecting data and predicting the approximate frequency given the probability.
        - Calculate theoretical probability
        - Calculate experimental probability

Other evidence:

- Through what other evidence (e.g., quizzes, tests, academic prompts, observations, homework, journals) will students demonstrate achievement of the desired results?
- How will students reflect upon and self-assess their learning?
- Are rubrics used to guide learners in self-assessment of progress toward mastery of the goal and to guide teachers in tracking student progress and providing feedback on progress toward accomplishing the goal?

4	Student attempted the problem and has the correct solution. All work is shown.
3	Student attempted the problem, but did not solve it correctly due to a minor computational error or a minor misunderstanding of the concept. Student used problem-solving strategies and all work is shown.
2	Student attempted the problem, but solved it incorrectly. Student showed perseverance by using problem-solving strategies and had a general understanding of the concept.
1	Student attempted the problem but did not use many problem-solving strategies. The student did not show evidence of perseverance and did not have a general understanding of the concept.
0	Student did not attempt the problem.

Test Item Specs

https://fsassessments.org/core/fileparse.php/3031/urlt/Grade-7-Math-Test-Item-Specifications1.pdf

Content Standard	***MAFS.7.SP*** *Statistics and Probability* ***MAFS.7.SP.3*** *Investigate chance processes and develop, use, and evaluate probability models.* ***MAFS.7.SP.3.7*** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. ***MAFS.7.SP.3.7a*** Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* ***MAFS.7.SP.3.7b*** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* Also Assesses: ***MAFS.7.SP.3.8*** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. ***MAFS.7.SP.3.8a*** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. ***MAFS.7.SP.3.8b*** Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. ***MAFS.7.SP.3.8c*** Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*
Assessment Limit	Numbers in items must be rational numbers.
Calculator	Yes
Item Type	Equation Editor GRID Matching Item Multiple Choice Multiselect Open Response Table Item
Context	Required

Sample Item	Item Type
A bag contains 3 red marbles and 6 blue marbles. What is the probability of randomly selecting a red marble from the bag?	Equation Editor
Tony has a bucket filled with 10 green, 3 blue, 1 red, and 7 yellow tennis balls. He removes 4 tennis balls from the bucket, without replacement. Which of the following outcomes could represent this selection? A. All of the tennis balls are blue. B. There is 1 tennis ball of each color. C. There are exactly 3 green tennis balls. D. There are more red tennis balls removed than other colors. E. The number of red tennis balls is the same as the number of blue tennis balls.	Multiselect
Select all situations that describe a probability of $\frac{1}{6}$ of drawing a red marble out of the bag. ☐ 1 red, 6 yellow, 6 green, 6 blue, 6 white ☐ 3 red, 4 yellow, 4 green, 4 blue, 3 white ☐ 4 red, 5 yellow, 5 green, 4 blue, 6 white ☐ 6 red, 6 yellow, 6 green, 6 blue, 6 white ☐ 6 red, 4 yellow, 8 green, 6 blue, 12 white	Multiselect
A bucket contains 5 green tennis balls and 2 yellow tennis balls. Tony removes 2 tennis balls, with replacement, from the bucket shown.  What is the probability that Tony will choose a yellow tennis ball and then a green tennis ball?	Equation Editor
A bucket contains 5 green tennis balls, 2 yellow tennis balls, and 6 red tennis balls. Tony removes 3 tennis balls, with replacement, from the bucket shown.  What is the probability that the first tennis ball is yellow, the second tennis ball is green, and the third tennis ball is red?	Equation Editor

Sample Item	Item Type
A bucket contains 5 green tennis balls, 2 yellow tennis balls, 6 red tennis balls, and 8 blue tennis balls. Tony removes 3 tennis balls, without replacement, from the bucket shown.  What is the probability that Tony removes 1 yellow, 1 green, and 2 blue tennis balls?	Equation Editor

Multiple Choice

A bag contains 4 red, 4 blue, 4 green, and 4 yellow marbles. A marble is randomly pulled from the bag and replaced seven times. The table shows the outcome of the experiment.

Trial	Outcome
1	Yellow
2	Blue
3	Yellow
4	Red
5	Blue
6	Yellow
7	Yellow

Which marble color's observed frequency is closest to its expected frequency?

Ⓐ red

Ⓑ blue

Ⓒ green

Ⓓ yellow

Multiselect

Tony has a bucket filled with green, blue, yellow, and red markers. He removes 3 markers from the bucket, with replacement.

Select all the outcomes that are possible.

☐ All of the markers removed are the same color.

☐ Only 1 marker of each color is removed.

☐ There are equal numbers of green and blue markers removed.

☐ There are equal numbers of red and yellow markers removed.

☐ There are only 2 colors of markers removed, and they were removed an equal number of times.

Learning Experiences

Plan learning experience(s) and instruction

Essential Understandings

What learning experiences and instruction will enable students to achieve the desired results?
How will the lesson design…

W = Help the students know **W**here the work is going and **W**hat is expected? Help the teacher know **W**here the students are coming from (prior knowledge, interests)?
H = **H**ook all students and hold their interest?
E = **E**quip students, help them **E**xperience the key ideas and Explore the issues?

Engagement Strategies

How do we assure the student engagement with the learning aligns with the cognitive complexity of the task?

- CRA Approach
  - Concrete (use of manipulatives)
  - Representational (Pictorial)
  - Abstract
- Model to Understand
  - Think Aloud
  - Graphic Organizer
- Discuss to Understand

R = Provide opportunities to **R**ethink and **R**evise their understanding and work?
E = Allow students to **E**valuate their work and its implications?

- Teach probability by posing the question, "What is the probability that if I roll a number cube labeled 1 through 6, it will land on a 1?" Allow students to respond on individual dry erase boards or whichever resource would allow the teacher to take a quick poll for understanding of basic probability.
- Teach probability by asking students to think about the terms "theory" and "probability" and record their own definitions on a piece of paper. Organize students into pairs. Allow them to discuss their definitions and make revisions (discussion may bring up concepts they actually know but didn't remember until their partner brought it up).
- Teach probability by modeling how to calculate theoretical and experimental probability.
- Task Analysis: Calculating Probability #1
    - Pass out number cubes (one per pair). They should record the outcomes of throwing a number dice 36 times.
    - Once everyone is finished, the teacher should use his/her data display to model to students how to calculate frequency of successful free throws as a fraction and percent. Students should independently attempt the same with his/her data.
    - Allow students to share out their findings. Invite students to explain how they arrived at their fractions and percent.
    - Facilitate a Think-Pair-Share with the following question: How does our theoretical data compare with our experimental data?
- Task Analysis: Calculating Probability #2
    - Pass out 3x5 index cards. Tell students to put his/her name on top. Tell them that you will pose a challenge question.
    - Give the students the following challenge question: "How many times out of 300 trials should I expect to toss a factor of a 12 on a number cube? Express this as a fraction and percent."
    - Go over the answer as a class, inviting students to share their thinking and defend their answers. It is suggested to use a random reporting method first and then invite participants to share.
        - Think-Pair-Share
        - Discussion Webs
        - Flexible Grouping (with guiding questions)
- Write to Understand
    - Think-Pair-Write-Share
    - Technology options

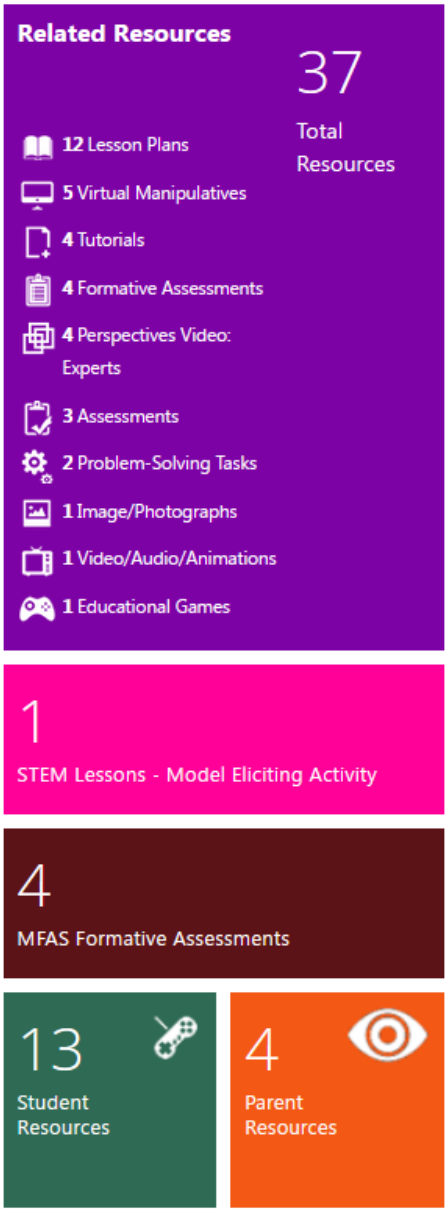
Resources

What is readily available or what do we need to plan ahead for to develop our learning experiences and instruction?

- **CPALMS Resources**
  http://www.cpalms.org/Public/PreviewStandard/Prev

Supports and Scaffolds

T = Be **T**ailored (personalized) to the different needs, interests, and abilities of learners?
O = Be **O**rganized to maximize initial and sustained engagement as well as effective learning?

iew/5486  **Example Task from Illustrative Math:** https://s3.amazonaws.com/illustrativemathematics/attachments/000/009/174/original/public_task_1216.pdf?1462395151	• Graphic organizers (Web) • Interactive Whiteboard and/or chart paper • Text-to-speech • Content delivered using multi-media (e.g., text, video, computer, etc.) • Prepared objects, pictures, words, or recorded communication supports to provide access to content and facilitate responding • Dichotomous questions that allow for making a choice of correct versus incorrect answers • Simpler or shorter problems • Word Splash (vocabulary strategy) • Multiplication table • Reference sheet (for formulas, conversions, algorithms, etc.) • Khan Academy Video: https://www.khanacademy.org/math/precalculus/prob-comb/basic-prob-precalc/v/basic-probability

For complete resources, link to:
CPALMS: http://www.cpalms.org
Access Weebly: https://accesstofls.weebly.com/math-resources.html
Test Item Specs: http://www.fldoe.org/accountability/assessments/k-12-student-assessment/end-of-course-eoc-assessments/test-item-specifications.stml
Learning Goals/Rubrics: https://www.leonschools.net/Page/27160